MATH 016 REVIEW II Discussions

[Run: 07/24/2020 at 18:3 Seed: 6541. Order of Checkable Items: Random.]

Rv **II-1.** On Tuesday your balance was six hundred three dollars and twenty-eight cents in the red and on Friday your balance was fifty-six dollars and three cents in the black. What is the signed number-phrase that represents the change in your balance from Tuesday to Friday?

Discussion: The *change* is the single action on the initial state that results in the final state.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - on Tuesday your balance was six hundred three dollars and twenty-eight cents in the red
  - on Friday your balance was fifty-six dollars and three cents in the black

  Since your balance has gone from the red to the black, from six hundred three dollars and twenty-eight cents in the red to fifty-six dollars and three cents in the black, this means that the action must have been a gain of six hundred three dollars and twenty-eight cents plus fty-six dollars and three cents, that is a gain of six hundred fifty-nine dollars and thirty one cents.
- In the *paper representation*, THEOREM 2 says that the *change* from an initial state to a final state is equal to the final state *ominus* the initial state. So we write the specifying-phrase

  $$+56.03 \textbf{ Dollars} \ominus -603.28 \textbf{ Dollars}$$

  that is

  $$+56.03 \textbf{ Dollars} \oplus +603.28 \textbf{ Dollars}$$

Either way, we end up writing

$$+659.31 \textbf{ Dollars}$$

Rv **II-2.** Your balance was seventy-six dollars and thirty-eight cents in the red and you made an eight hundred seventy-six dollars and eleven cents deposit. What is the signed number-phrase that represents your new balance?

Discussion: The *final* state is the result of the *action* on the *initial* state.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - The initial state of your account was seventy-six dollars and thirty-eight cents in the red
  - The action on this initial state was an eight hundred and seventy-six dollars and eleven cents deposit.

  Since your are *depositing* money from an account that was already in the *red*, the eight hundred and seventy-six dollars and eleven cents first go to the seventy-six dollars and thirty-eight cents in the red to give a final balance of *seven hundred ninety-nine dollars and seventy-three cents in the black*
- In the *paper representation*, we write the signed specifying-phrase
$$-76.38\ \textsf{Dollars} \oplus +876.11\ \textsf{Dollars}$$
and we identify it.

Either way, we end up writing

$$+799.73\ \textsf{Dollars}$$

Rv **II-3.** Execute the specifying-phrase $+837.44 \ominus +869.04$

Discussion: REVIEWdiscussion

Rv **II-4.** Execute the specifying-phrase $[-5\ \textsf{Carrots}] \times \left[+7\ \frac{\textsf{Cents}}{\textsf{Carrot}}\right]$

Discussion: We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - We have five carrots disappearing from the warehouse
  - These carrots were bad carrots and would have cost seven cents per carrot to get rid of.

  Altogether then, this is going to be a gain of thirty-five cents for the business.
- In the *paper representation*, we *co-multiply*:
  i. we multiply the *denominators* (with cancellation):
  $$\cancel{\textsf{Carrots}} \times \frac{\textsf{Cents}}{\cancel{\textsf{Carrot}}} = \textsf{Cents}$$
  ii. we multiply the sizes of the numerators
  $$5 \times 7 = 35$$
  iii. we multiply the signs of the numerators
  $$(-) \otimes (-) \text{ gives } (+)$$

Either way, we have identified the specifying-phrase $[-5\ \textsf{Carrots}] \times \left[-7\ \frac{\textsf{Cents}}{\textsf{Carrot}}\right]$ as

+35 **Cents**

Rv **II-5.** Execute $-53-(-21)$

Discussion: REVIEWdiscussion

Rv **II-6.** Execute $56+13-(-7)+31$

Discussion: REVIEWdiscussion

Rv **II-7.** Execute the specifying-phrase $+792.037 \oplus -834.28$

Discussion: REVIEWdiscussion

Rv **II-8.** Execute $0 \div -45$

Discussion: REVIEWdiscussion

Rv **II-9.** Execute $2-1+4-1-3+5-3-2+1+6-1+5+2$

Discussion:

i. The symbol $\oplus$ goes without saying,

ii. The symbols $+$ and $-$ are the signs of the signed numerators,

iii. If the first numerator has no sign, the sign $+$ goes without saying.

$2-1+4-1-3+5-3-2+1+6-1+5+2$

$$\underbrace{+2 \oplus -1}$$
$$\underbrace{+1 \oplus +4}$$
$$\underbrace{-3 \oplus -1}$$
$$\underbrace{-4 \oplus -3}$$
$$\underbrace{-7 \oplus +5}$$
$$\underbrace{-2 \oplus -3}$$
$$\underbrace{-5 \oplus -2}$$
$$\underbrace{-7 \oplus +1}$$
$$\underbrace{-6 \oplus +6}$$
$$\underbrace{0 \oplus -1}$$
$$\underbrace{-1 \oplus +5}$$
$$\underbrace{+4 \oplus +2}$$
$$+6$$

Rv **II-10.** Your balance was seventy-six dollars and thirty-eight cents in the red and you made an eight hundred seventy-six dollars and eleven cents withdrawal. What is the signed number-phrase that represents your new balance?

Discussion: The *final* state is the result of the *action* on the *initial* state.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - The initial state of your account was seventy-six dollars and thirty-eight cents in the red
  - The action on this initial state was an eight hundred and seventy-six dollars and eleven cents withdrawal.

  Since your are *withdrawing* money from an account that was already in the *red*, the eight hundred and seventy-six dollars and eleven cents add to the seventy-six dollars and thirty-eight cents to give a final balance of *nine hundred fifty-two dollars and forty-nine cents in the black*.
- In the *paper representation*, we write the signed specifying-phrase
$$-76.38\ \textsf{Dollars} \oplus -876.11\ \textsf{Dollars}$$
and we identify it.

Either way, we end up writing

$$+952.49\ \textsf{Dollars}$$

Rv **II-11.** Execute the specifying-phrase $[+4\ \textsf{Apples}] \times \left[-2\ \frac{\textsf{Dimes}}{\textsf{Apple}}\right]$

Discussion: We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - We have four apples appearing into the warehouse
  - Each of these apples are bad apples and will cost two dimes per apple to get rid of.

  Altogether then, this is going to cost eight dimes to the business.
- In the *paper representation*, we *co-multiply*:

  i. we multiply the *denominators* (with cancellation):
$$\cancel{\textsf{Apples}} \times \frac{\textsf{Dimes}}{\cancel{\textsf{Apple}}} = \textsf{Dimes}$$
  ii. we multiply the sizes of the numerators
$$4 \times 2 = 8$$
  iii. we multiply the signs of the numerators
$$(+) \otimes (-) \text{ gives } (-)$$

Either way, we have identified the specifying-phrase $[+4\ \textsf{Apples}]\times\left[-2\ \frac{\textsf{Dimes}}{\textsf{Apple}}\right]$ as

$$-8\ \textsf{Dimes}$$

Rv **II-12.** Execute the specifying-phrase $-234.938 \ominus -402.772$

Discussion: REVIEWdiscussion

Rv **II-13.** Execute $+2-1+4-1-3+5-3-2+1+6-1+5+2$

Discussion:

i. The symbol $\oplus$ goes without saying,

ii. The symbols $+$ and $-$ are the signs of the signed numerators,

iii. If the first numerator has no sign, the sign $+$ goes without saying.

$+2-1+4-1-3+5-3-2+1+6-1+5+2$

$$\begin{array}{l}
\underbrace{+2\oplus -1}\\
\quad\underbrace{+1\oplus +4}\\
\qquad\underbrace{-3\oplus -1}\\
\qquad\quad\underbrace{-4\oplus -3}\\
\qquad\qquad\underbrace{-7\oplus +5}\\
\qquad\qquad\quad\underbrace{-2\oplus -3}\\
\qquad\qquad\qquad\underbrace{-5\oplus -2}\\
\qquad\qquad\qquad\quad\underbrace{-7\oplus +1}\\
\qquad\qquad\qquad\qquad\underbrace{-6\oplus +6}\\
\qquad\qquad\qquad\qquad\quad\underbrace{0\oplus -1}\\
\qquad\qquad\qquad\qquad\qquad\underbrace{-1\oplus +5}\\
\qquad\qquad\qquad\qquad\qquad\quad\underbrace{+4\oplus +2}\\
\qquad\qquad\qquad\qquad\qquad\qquad +6
\end{array}$$

Rv **II-14.** Given the problem in **Dollars**

$$x < -371.45$$

what is the *graph* of its solution subset?

Discussion: This inequation lets IN all the numbers that are *smaller* than -371.45.

The inequation is *strict* so that it leaves OUT the *boundary point* -371.45.

The *graph* of the solution subset is therefore:

Rv **II-15.** Given the following "events"

Jack's "event" $= [-4\ \textsf{Apples}] \times \left[+6\ \frac{\textsf{Dimes}}{\textsf{Apple}}\right]$

and

Jill's "event" $= [-5\ \textsf{Bananas}] \times \left[-3\ \frac{\textsf{Dimes}}{\textsf{Banana}}\right]$,

identify the specifying-phrase Jack's "event" $\oplus$ Jill's "event'.

Discussion: REVIEWdiscussion

Rv **II-16.** Your balance was seventy-six dollars and thirty-eight cents in the black and you made an eight hundred seventy-six dollars and eleven cents withdrawal. What is the signed number-phrase that represents your new balance?

Discussion: The *final* state is the result of the *action* on the *initial* state.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - The initial state of your account was seventy-six dollars and thirty-eight cents in the black
  - The action on this initial state was an eight hundred and seventy-six dollars and eleven cents withdrawal.

  Since your are *withdrawing* more money than was in the account, the eight hundred and seventy-six dollars and eleven cents break down to the seventy-six dollars and thirty-eight cents that were in the account and the remainder that gives a final balance of *seven hundred ninety-nine dollars and seventy-three cents in the red*.
- In the *paper representation*, we write the signed specifying-phrase

  $$+76.38\ \textsf{Dollars} \oplus -876.11\ \textsf{Dollars}$$

  and we identify it.

Either way, we end up writing

$$-799.73 \text{ Dollars}$$

Rv **II-17.** Given the problem in **Dollars**

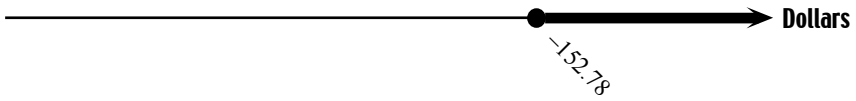
$$x \geqq -152.78$$

what is the *graph* of its solution subset?

Discussion: This inequation lets IN all the numbers that are *larger* than -152.78.

The inequation is *lenient* so that it lets IN the *boundary point* -152.78.

The *graph* of the solution subset is therefore:



Rv **II-18.** Execute $31 \div 0$

Discussion: REVIEWdiscussion

Rv **II-19.** Execute the specifying-phrase $-62.394 \oplus +39.977$

Discussion: REVIEWdiscussion

Rv **II-20.** You thought your balance was one hundred seventy-two dollars and fifty-seven cents in the black but you just found out that a twelve dollars and fifty-six cents check you had deposited bounced. What is the signed number-phrase that represents your new balance?

Discussion: *Removing* a deposit or *removing* a withdrawal is a real-world *action* that is represented on paper by a *subtraction*.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - You thought the balance was one hundred seventy-two dollars and fifty-seven cents in the black
  - but this balance included a twelve dollars and fifty-six cents check

  Since the check bounced, the balance is actually twelve dollars and fifty-six cents less than you thought, that is one hundred sixty dollars and one cent in the black.
- In the *paper representation*, we write the specifying-phrase

$$+172.57 \text{ Dollars} \ominus +12.56 \text{ Dollars}$$

which we identify by *adding the opposite of* the second number-phrase to the first number-phrase

$$+172.57\ \textsf{Dollars} \oplus -12.56\ \textsf{Dollars}$$

Either way, we end up writing the signed number-phrase

$$+160.01\ \textsf{Dollars}$$

Rv **II-21.** You thought your balance was one hundred seventy-two dollars and fifty-seven cents in the red but you just found out that an unjustified twelve dollars and fifty-six cents charge has been removed. What is the signed number-phrase that represents your new balance?

Discussion: *Removing* a deposit or *removing* a withdrawal is a real-world *action* that is represented on paper by a *subtraction*.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - You thought the balance was one hundred seventy-two dollars and fifty-seven cents in the red
  - but this balance included a twelve dollars and fifty-six cents charge

  Since the charge was removed, the balance is actually twelve dollars and fifty-six cents more than you thought, that is one hundred sixty dollars and one cent in the red.
- In the *paper representation*, we write the specifying-phrase

  $$-172.57\ \textsf{Dollars} \ominus -12.56\ \textsf{Dollars}$$

  which we identify by *adding the opposite of* the second number-phrase to the first number-phrase

  $$-172.57\ \textsf{Dollars} \oplus +12.56\ \textsf{Dollars}$$

Either way, we end up writing the signed number-phrase

$$-160.01\ \textsf{Dollars}$$

Rv **II-22.** Your balance was seventy-six dollars and thirty-eight cents in the black and you made an eight hundred seventy-six dollars and eleven cents deposit. What is the signed number-phrase that represents your new balance?

Discussion: The *final* state is the result of the *action* on the *initial* state.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,

  - The initial state of your account was seventy-six dollars and thirty-eight cents in the black
  - The action on this initial state was an eight hundred and seventy-six dollars and eleven cents deposit.

  Since your are *depositing* money on an account that was already in the *black*, the eight hundred and seventy-six dollars and eleven cents add to the seventy-six dollars and thirty-eight cents to give a final balance of *nine hundred fifty-two dollars and forty-nine cents in the black*.
- In the *paper representation*, we write the signed specifying-phrase

$$+76.38\ \textsf{Dollars} \oplus +876.11\ \textsf{Dollars}$$

  and we identify it.

Either way, we end up writing

$$+952.49\ \textsf{Dollars}$$

Rv **II-23.** On Monday your balance was three hundred thirty-two dollars and seventy one cents in the red and on Thursday your balance was seventy-four dollars and forty-six cents in the red. What is the signed number-phrase that represents the change in your balance from Monday to Thursday?

Discussion: The *change* is the single action on the initial state that results in the final state.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - on Monday your balance was three hundred thirty-two dollars and seventy one cents in the red
  - on Thursday your balance was seventy-four dollars and forty-six dollars in the red

  Since, while still in the red, the balance has gone *down in size*, from three hundred thirty-two dollars and seventy-one cents to seventy-four dollars and forty-six dollars, this means that the action must have been a gain of two hundred fifty-eight dollars and twenty-five cents.
- In the *paper representation*, THEOREM 2 says that the *change* from an initial state to a final state is equal to the final state *ominus* the initial state. So we write the specifying-phrase

$$-74.46\ \textsf{Dollars} \ominus -332.71\ \textsf{Dollars}$$

  that is

$$-74.46\ \textsf{Dollars} \oplus +332.71\ \textsf{Dollars}$$

Either way, we end up writing

$$-258.25 \text{ Dollars}$$

Rv **II-24.** Execute for *plain numbers*: $8 - 13$

Discussion: REVIEWdiscussion

Rv **II-25.** Given the data set

-3.2 **Dollars**, -2.6 **Dollars**, -1.3 **Dollars**, $+0.7$ **Dollars**, $+1.4$ **Dollars**, $+2.6$ **Dollars**, $+3.1$ **Dollars**

and the formula in **Dollars**

$$x < +3.2$$

What are the solutions in **Dollars**?

Discussion: REVIEWdiscussion