Notes from the Mathematical Underground

Edited by

Alain Schremmer.
Mathematics Department, Community College of Philadelphia,
1700 Spring Garden Street
Philadelphia, PA 19130

There is a widespread belief that, in order to proceed into mathematics, all that is needed is a certain number of prerequisite skills, collectively going by the name of Basic Algebra. Hence, since those lacking these skills are supposedly barred from mathematics, the need for remediation. What is conspicuously absent from this *Weltanschauung* is summarized by the very first sentence in Bourbaki's *Elements de Mathématique*: There is no prerequisite, only a measure of mathematical maturity.

But then, whether most mathematics remediation is a failure, as (Steen, 1991) maintained, and (Laughbaum, 1992) concurred with and (Rotman, 1993) disagreed with in these pages, must surely be an ill-posed problem. Laughbaum's opening paragraph mentions that "*it is difficult for instructors to spend adequate time on fundamental concepts*" and gives as an example student performance on factoring, one of the "*traditionally taught skills at the remedial level*". He blames "*symbol manipulation*" and his solution is to use graphics calculator and to bend the curriculum to fit. On the other hand, Rotman's "*own developmental program is quite successful by traditional standards*" and what he advocates is the creation of a task force! Just in case, one supposes.

What are *we* to conclude from all this? A clue is that, if both authors mentioned *topics*, mostly topics in algebra, neither gave any *reason* for learning these particular topics. In reading their "viewpoints", one cannot avoid the impression that God created The Curriculum, probably some time before the Big Bang, even if, deplorably, She didn't have the foresight to create students good enough to learn it from us. Women! Usually, there is a vague, unspoken assumption that it is somehow "useful" for the students. But, surely, as a tool for "applications", basic algebra just won't do. In fact, even "*First Semester Calculus has* no *applications*" as (Dudley, 1988) pointed out as Conclusion #5 in an article on calculus texts. When confronted with this harsh reality, we usually fall back on something like factoring being good for the students' soul.

The matter must depend on our idea of what mathematics *is* and of what possible *use* it can be to "just plain folk" (Goldstein, 1986). In other words, this raises the question of what *learning* mathematics consists of. Of course, none of the above viewpoints saw fit to disclose any *idea* on that matter. So what *is* mathematics? And what, therefore, should Developmental Courses develop in order for just plain folk to learn mathematics?

As an example, I propose to specify Basic Algebra "equationally" rather than "descriptively" or "prescriptively" namely as the very least needed to deal with the "elementary" initial value problems $f\,'(x) = f(x)$ and $f\,''(x) = \pm\, f(x)$. We argued in (Schremmer & Schremmer, 1989) that the Precalculus and the Differential Calculus could be integrated into a systematic study of functions culminating with the "elementary" functions based on their (Laurent) polynomial approximations. This approach, going back to (Lagrange, 1797) and which we expounded in (Schremmer & Schremmer, 1990), conceptually requires very little beyond familiarity with decimal numbers. Reverse engineering then determines the required Basic Algebra. For instance, we need to divide in ascending as well as in descending powers because while, near ∞, $\dfrac{x^3 - 1}{(x+1)^2} = +x - 2 + \dfrac{3}{x} + (\ldots)$, near 0, $\dfrac{-1 + h^3}{(+1 + h)^2} = -1 + 2h - 3h^2 + (\ldots)$. (Note that we know when to stop the division: when the quotient has *concavity*.)

Thus, after Basic Algebra, considered as remediation, students can reach First Semester Calculus level in two semesters. Moreover, observe that, if the above definition of differential calculus meets the challenge implicitly posed by Dudley's Conclusion #5, it also has the merit to prepare for an alternative to the traditional Second Semester Calculus better suited for students *not* headed towards Physics, Engineering or Mathematics, namely a course in Dynamical Systems. I will expand on this in a future column and, for now, suffice it to say that reviewers of an—otherwise unsuccessful—NSF proposal were quite taken by the idea.

• • •

Starting from the fact that "*the very nature of number is that number is* '*unitless*'", Reader Laurie Golson raises several issues concerning my advocating the use of units. She also indirectly showed how inadequate my presentation was. For instance, I should have pointed out that the unit for the 2 in "2 times 25 strawberries" is "25 strawberries" as, "times" notwithstanding, this is an *additive power* rather than a *multiplication*. Bad language always creates problems. Still, what *is* a number? Since the notion has evolved throughout history, asking what it should be in a given course is certainly a most important question. If readership interest warrants it, we could give space to a forum.

• • •

When, a *very* long time ago, I was first presented with the dictum that "minus times a minus is a plus", I of course firmly rejected it as obviously false. Upon consideration however, being a nice middle class child with no particular problem, I decided to *believe* my teacher and to memorize "minus times a minus is a plus" over my own better judgment, knowing full well that this was the only way to the future. You could say that this is when I became schizophrenic or, at the very least, when I learned that to succeed requires being dishonest.

In my previous column, I had held that "the main problem students have with mathematics [is that] the conventional curriculum makes it completely impossible for them to see […] the overall architecture according to which these things hang together. For instance, the problem in the conventional approach to differential calculus is that limits, continuity, differentiability are introduced in the first few weeks of the course so that, if those concepts are not mastered immediately—and they cannot in such an architecture, it is impossible for the students to function intelligently: All they can do, all they must do, is to believe, memorize and become schizoprenic if not learn being dishonest.

To an extent, four-year schools can get away with it because *they* have enough "good" students, that is students whose social background is such that they have no reason not to trust their instructor. After all, they probably belong to the same social class. By and large however, *our* students are in a different situation as *they* have little ground to trust us or a societal system that is grinding them down. So, even though they have been brainwashed into truly believing that learning equals memorizing, they run into the unfortunate problem that one cannot memorize when in a state of anxiety, mathematical or otherwise.

Thus, it is indeed the very lack of architecture of the conventional approach that is a barrier to *our* students and I would propose that we discuss architectures. For example, if nothing else, the architecture just alluded to above has the merit to leave enough time to "*spend adequate time on fundamental concepts*", be it in the course of remediation or in that of the differential calculus. But there are, of course, other architectures for which however I would not be a good advocate and advocates of such architectures should use this column.

Speaking of architecture, or rather, the lack thereof, could this be the result of the modern trend to find safety in numbers? It indeed used to be that a large number of reviewers was necessary to ensure the salability of the fat calculus text: Presumably, at least the favorable reviewers would use it. Moreover, and to quote Dudley again: "*If one* [reviewer] *writes that the author has left out the* $\tan(x/2)$ *substitution in the section on techniques of integration, how can he or she do that, we won't be able to integrate* $3/(4 + 5 \sin 6x)$, *how can anyone claim to know calculus who can't do that.*" But now it seems that it is a large number of *authors* that has become necessary in addition to "*the generous support of the National Science Foundation*" mentioned in (Hughes-Hallett et al., 1994), apparently the number one seller and a real critique of which is, I think, vastly overdue. It certainly does not have much of an architecture: It begins with Chapter 1 - A Library of Functions, Chapter 2 - Key Concept: The derivative. This under the name, inter alia, of someone who once wrote that Calculus "*frequently hurries into such questions as differentiation and integration, and often fails to put the proper emphasis on what the subject is all about, namely function of a real variable*" (Gleason, 1967). And then, all of this in 148 pages! I can well understand why the required background is left rather fuzzy: "*We have found that this curriculum to be thought-provoking for well-prepared students while still accessible to students with weak algebra backgrounds.*" Presumably, *their* students "think" about calculus while *our* students can only be expected to "access" it.

As long as we in two-year colleges allow ourselves to be driven by four-year schools, we are doomed to impotence. It is interesting in this respect that even Dudley should have about concluded his article with the statement that "*Calculus is a splendid screen for screening out dummies, but it also screens out perfectly intelligent people who find it difficult to deal with quantities.*" Can there really be such people or is this a convenient way to dispose of them properly?

A tidbit about architecture. On page 967 of his Calculus (Anton, 1988), the famous author of *In Defense of the Fat Calculus Text* (Anton, 1991) essentially defines a differentiable function of *two* variables as a function that can be approximated by an affine function. Of course, on page 150, he had begun by saying that a function of *one* variable is differentiable if it has a derivative.

References

Anton, H. (1988). *Calculus.* New York: Wiley.

Anton, H. (1991). In Defense of the Fat Calculus Text. *UME Trend, 2*(6), 1.

Dudley, U. (1988). Review of "Calculus with Analytic Geometry" by George F. Simmons. *The American Mathematical Monthly, 95*(9), 888-892.

Gleason, A. (1967,). *The Geometric Content of Advanced Calculus.* Paper presented at the CUPM Geometry Conference.

Goldstein, J. (1986,). *Calculus Syllaby, Report of the Content Workshop.* Paper presented at the Toward a Lean and Lively Calculus, Tulane.

Hughes-Hallett, D., Gleason, A. M., Flath, D. E., Gordon, S. P., Lomen, D. O., Lovelock, D., McCallum, W. G., Osgood, B. G., Pasquale, A., Tecosky-Feldman, J., Thrash, J. B., Thrash, K. R., Tucker, T. W., & Bretscher, O. K. (1994). *Calculus.* New York: Wiley.

Lagrange, J. L. (1797). *Théorie des Fonctions Analytiques.* Paris: Gauthier-Villars.

Laughbaum, E. D. (1992). A Time for Change in Remedial Mathematics. *The AMATYC Review, 13*(2), 7-10.

Rotman, J. W. (1993). Time, Indeed, for a Change in Developmental Mathematics. *The AMATYC Review, 16*(1), 8-11.

Schremmer, F., & Schremmer, A. (1989). Integrated Precalculus and Differential Calculus: A Lagrangian Approach. *The AMATYC Review, 11*(1, Part 2), 28-31.

Schremmer, F., & Schremmer, A. (1990). An Introduction to Lagrangian Differential Calculus. *The AMATYC Review, 11*(2), 16-25.

Steen, L. A. (1991,). *Twenty Questions for Computer Reformers.* Paper presented at the Second Anual Conference on Technology in Collegiate Mathematics.