I-1. Let $f$ be the function specified by the *quantitative bounded graph*

For which input(s), if any, will $f$ return the output 0?

**Discussion:**

i. We mark the output 0 on the output ruler,

ii. We draw the output level line through the *output 0*

iii. We mark with plot point(s) the intersection(s), if any, of the output level line with the graph

iv. We draw an input level line through each of the plot point(s),

v. Then, the input number(s) is/are where the input level line(s) intersect(s) the input ruler.
I-2. Let \( f \) be the function specified by the global input-output rule
\[
x \xrightarrow{f} f(x) = (20.04)x^5
\]
Find the local graph of \( f \) near \( \infty \)

**Discussion:**

i. We normalize the global input output rule:
\[
x \xrightarrow{f} f(x) = (+\text{bounded})x^{+\text{odd}}
\]
\[
= (+\text{bounded}) \cdot (x) \cdot \ldots \cdot (x)
\]
odd number of copies of \( x \)

ii. To localize near \( \infty \), we compute the output for inputs that are \( \pm\text{large} \):

\[
+\text{large} \xrightarrow{f} f(+\text{large}) = (+\text{bounded})(+\text{large})^{+\text{odd}}
\]
\[
= (+\text{bounded}) \cdot (+\text{large}) \cdot \ldots \cdot (+\text{large})
\]
odd number of copies of \( +\text{large} \)
\[
= +\text{large}
\]

and

\[
-\text{large} \xrightarrow{f} f(-\text{large}) = (+\text{bounded})(-\text{large})^{-\text{odd}}
\]
\[
= (+\text{bounded}) \cdot (-\text{large}) \cdot \ldots \cdot (-\text{large})
\]
odd number of copies of \( -\text{large} \)
\[
= -\text{large}
\]

I-3. Let \( f \) be the function specified by the global input-output rule
\[
x \xrightarrow{f} f(x) = (38.48)x^{-6}
\]
Find the local graph of \( f \) near \( \infty \)

**Discussion:**
i. We normalize the global input output rule:

\[ x \xrightarrow{f} f(x) = (+\text{bounded}) x^{-\text{even}} \]

ii. To localize near \( \infty \), we compute the output for inputs that are \( \pm \text{large} \):

\[
\begin{align*}
+\text{large} & \xrightarrow{f} f(+\text{large}) = (+\text{bounded})(+\text{large})^{-\text{even}} \\
& = (+\text{bounded}) \cdot \ldots \cdot (+\text{large}) \\
& \text{even number of copies of } +\text{large} \\
& = +\text{small}
\end{align*}
\]

and

\[
\begin{align*}
-\text{large} & \xrightarrow{f} f(-\text{large}) = (+\text{bounded})(-\text{large})^{-\text{even}} \\
& = (+\text{bounded}) \cdot \ldots \cdot (-\text{large}) \\
& \text{even number of copies of } -\text{large} \\
& = +\text{small}
\end{align*}
\]

iii. Output Ruler

Input Ruler

Screen

Offscreen

((

0

(4

I-4. Let \( f \) be the function specified by the global input-output rule

\[ x \xrightarrow{f} f(x) = (+35.09)x^{+5} \]

Find the local graph of \( f \) near 0.

Discussion:

i. We normalize the global input output rule:

\[ x \xrightarrow{f} f(x) = (+\text{bounded}) x^{+\text{odd}} \]

ii. To localize near 0, we compute the output for inputs that are \( \pm \text{small} \):
I-5. Let $f$ be the function specified by the global input-output rule

$$x \xrightarrow{f} f(x) = (-49.74)x^{-7}$$

Find the local graph of $f$ near 0

**Discussion:**

i. We normalize the global input output rule:

$$x \xrightarrow{f} f(x) = (-\text{bounded})x^{-\text{odd}}$$

ii. To localize near 0, we compute the output for inputs that are $\pm\text{small}$
I-6. Let $f$ be the function specified by the global input-output rule

$$x \xrightarrow{f} f(x) = (95.19)x^+6$$

Find Height-sign $f|_{\text{near } \infty}$

**Discussion:**

i. We normalize the global input output rule:

$$x \xrightarrow{f} f(x) = (+\text{bounded})x^+\text{even}$$

ii. To localize near $\infty$, we compute the output for inputs that are $\pm\text{large}$.
\[ +\text{large} \xrightarrow{f} f(+\text{large}) = (+\text{bounded})(+\text{large})^{+\text{even}} \]
\[ = (+\text{bounded}) \cdot (+\text{large}) \cdot \ldots \cdot (+\text{large}) \]
\[ = +\text{large} \]

and

\[ -\text{large} \xrightarrow{f} f(-\text{large}) = (+\text{bounded})(-\text{large})^{+\text{even}} \]
\[ = (+\text{bounded}) \cdot (-\text{large}) \cdot \ldots \cdot (-\text{large}) \]
\[ = +\text{large} \]

iii. 

iv. Height-sign \( f \) near \( \infty \) = (+, +) 
(As seen from \( \infty \))

I-7. Let \( f \) be the function specified by the global input-output rule

\[ x \xrightarrow{f} f(x) = (-65.52)x^{-5} \]

Find Height-sign \( f \) near 0

Discussion:

i. We normalize the global input output rule:

\[ x \xrightarrow{f} f(x) = (-\text{bounded})x^{-\text{odd}} \]

ii. To localize near 0, we compute the output for inputs that are ±small
Let \( f \) be the function specified by the global input-output rule

\[
x \xrightarrow{f} f(x) = (-29.73)x^{-4}
\]

Find Slope-sign \( f \) near \( \infty \).

**Discussion:**

i. We normalize the global input output rule:

\[
x \xrightarrow{f} f(x) = (-\text{bounded})x^{-\text{even}}
\]

ii. To localize near \( \infty \), we compute the output for inputs that are \( \pm \text{large} \):
\[ +\text{large} \xrightarrow{f} f(+\text{large}) = (-\text{bounded})(+\text{large})^{-\text{even}} \]
\[ = -\text{bounded} \]
\[ = (-\text{large}) \cdot \ldots \cdot (+\text{large}) \]
\[ \text{even number of copies of } +\text{large} \]
\[ = -\text{small} \]

and

\[ -\text{large} \xrightarrow{f} f(-\text{large}) = (-\text{bounded})(-\text{large})^{-\text{even}} \]
\[ = -\text{bounded} \]
\[ = (-\text{large}) \cdot \ldots \cdot (-\text{large}) \]
\[ \text{even number of copies of } -\text{large} \]
\[ = -\text{small} \]

iii. Slope-sign \( f \big|_{\text{near } \infty} = (\diagup, \\diagdown) \)  
(As seen from \( \infty \))

iv. Slope-sign \( f \big|_{\text{use } 0} \)

I-9. Let \( f \) be the function specified by the global input-output rule
\[ x \xrightarrow{f} f(x) = (-18.83)x^{+6} \]

Find Slope-sign \( f \big|_{\text{near } 0} \)

Discussion:

i. We normalize the global input output rule:
\[ x \xrightarrow{f} f(x) = (-\text{bounded})x^{+\text{even}} \]

ii. To localize near \( 0 \), we compute the output for inputs that are \( \pm \text{small} \)
\[ +\text{small} \xrightarrow{f} f(+\text{small}) = (-\text{bounded})(+\text{small})^{+\text{even}} = (-\text{bounded}) \cdot (+\text{small}) \cdot \ldots \cdot (+\text{small}) \]

where \( \text{even} \) number of copies of \(+\text{small}\)

\[ = -\text{small} \]

\[ -\text{small} \xrightarrow{f} f(-\text{small}) = (-\text{bounded})(-\text{small})^{+\text{even}} = (-\text{bounded}) \cdot (-\text{small}) \cdot \ldots \cdot (-\text{small}) \]

where \( \text{even} \) number of copies of \(-\text{small}\)

\[ = -\text{small} \]

iii. \( \iota \)

iv. \( \iota \) \( \text{Slope-sgn}\) \( f \) \text{near} \( 0 \) \( = (\bigvee, \bigwedge) \)

I-10. Let \( f \) be the function specified by the global input-output rule

\[ x \xrightarrow{f} f(x) = (-73.49)x^{-5} \]

Find Concavity-sgn \( f \) \text{near} \( \infty \).

Discussion:

i. We normalize the global input output rule:

\[ x \xrightarrow{f} f(x) = (-\text{bounded})x^{-\text{odd}} \]

ii. To localize near \( \infty \), we compute the output for inputs that are \( \pm\text{large} \):
\[ +\text{large} \xrightarrow{f} f(+\text{large}) = (-\text{bounded})(+\text{large})^{-\text{odd}} \]
\[ = -\text{bounded} \]
\[ = \frac{(+\text{large}) \cdot \ldots \cdot (+\text{large})}{\text{odd number of copies of } +\text{large}} \]
\[ = -\text{small} \]

and

\[ -\text{large} \xrightarrow{f} f(-\text{large}) = (-\text{bounded})(-\text{large})^{-\text{odd}} \]
\[ = -\text{bounded} \]
\[ = \frac{(-\text{large}) \cdot \ldots \cdot (-\text{large})}{\text{odd number of copies of } -\text{large}} \]
\[ = +\text{small} \]

iii. \( \infty \)

iv. Concavity-sign \( f \) near \( \infty = (\cap, \cup) \)
(As seen from \( \infty \))

I-11. Let \( f \) be the function specified by the global input-output rule
\[ x \xrightarrow{f} f(x) = (+66.23)x^3 \]
Find Concavity-sign \( f \) near 0

Discussion:

i. We normalize the global input output rule:
\[ x \xrightarrow{f} f(x) = (+\text{bounded})x^{+\text{odd}} \]

ii. To localize near 0, we compute the output for inputs that are \( \pm \text{small} \):
\[ +\text{small} \xrightarrow{f} f(+\text{small}) = (+\text{bounded})(+\text{small})^{+\text{odd}} \]
\[ = (+\text{bounded}) \cdot (+\text{small}) \cdot \ldots \cdot (+\text{small}) \]
\[ = +\text{small} \]

and

\[ -\text{small} \xrightarrow{f} f(-\text{small}) = (+\text{bounded})(-\text{small})^{+\text{odd}} \]
\[ = (+\text{bounded}) \cdot (-\text{small}) \cdot \ldots \cdot (-\text{small}) \]
\[ = -\text{small} \]

iii. 

iv. Concavity-sign \( f \) near 0 = \( \cap, \cup \)

I-12. Let \( f \) be a power function whose local graph near \( \infty \) is

Find the local graph of \( f \) near 0.

Discussion:

i. We read the features of \( f \) from the local graph near \( \infty \) and write the corresponding features of the global input-output rule:
Features of the Local Graph near $\infty$ | Features of the Global Input-Output Rule
--- | ---
$\text{large} \xrightarrow{f} f(\text{large}) = \text{small}$ | The exponent must be $-$
$+ \xrightarrow{f} f(+) = -$ | The coefficient must be $-$
$- \xrightarrow{f} f(-) = +$ | Since the coefficient is $-$, the exponent must be odd

The global input-output rule of $f$ is therefore of the form:

$$x \xrightarrow{f} f(x) = (-\text{bounded})x^{-\text{odd}}$$

**ii.** To *localize* near 0, we compute the output for inputs that are $\pm \text{small}$

$$+\text{small} \xrightarrow{f} f(+) = (-\text{bounded})(+\text{small})^{-\text{odd}}$$

$$= \frac{-\text{bounded}}{(+\text{small}) \cdot \ldots \cdot (+\text{small})}$$

odd number of copies of $+\text{small}$

$$= -\text{large}$$

and

$$-\text{small} \xrightarrow{f} f(-) = (-\text{bounded})(-\text{small})^{-\text{even}}$$

$$= \frac{-\text{bounded}}{(-\text{small}) \cdot \ldots \cdot (-\text{small})}$$

even number of copies of $-\text{small}$

$$= +\text{large}$$

**iii.**

I-13. Let $f$ be a *power* function whose local graph near 0 is
Find the local graph of \( f \) near \( \infty \).

**Discussion:**

i. We read the features of \( f \) from the local graph near 0 and write the corresponding features of the global input-output rule:

<table>
<thead>
<tr>
<th>Features of the Local Graph near 0</th>
<th>Features of the Global Input-Output Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>small ( \xrightarrow{f} f(\text{small}) = \text{small} )</td>
<td>The exponent must be +</td>
</tr>
<tr>
<td>( + \xrightarrow{f} f(+) = + )</td>
<td>The coefficient must be +</td>
</tr>
<tr>
<td>( - \xrightarrow{f} f(-) = - )</td>
<td>Since the coefficient is +, the exponent must be odd</td>
</tr>
</tbody>
</table>

The global input-output rule of \( f \) must therefore be of the form:

\[
x \xrightarrow{f} f(x) = (\text{bounded})x^{\text{odd}}
\]

ii. To localize near \( \infty \), we compute the output for inputs that are \( \pm \text{large} \):

\[
+\text{large} \xrightarrow{f} f(\text{large}) = (\text{bounded})(+\text{large})^{\text{odd}}
\]

\[
= (\text{bounded}) \cdot (+\text{large}) \cdot \ldots \cdot (+\text{large})
\]

\[
= +\text{large}
\]

and

\[
-\text{large} \xrightarrow{f} f(-\text{large}) = (+\text{bounded})(-\text{large})^{\text{odd}}
\]

\[
= (+\text{bounded}) \cdot (-\text{large}) \cdot \ldots \cdot (-\text{large})
\]

\[
= -\text{large}
\]
I-14. Let $f$ be a power function whose local graph near $-\infty$ is

Find the local graph of $f$ near $0^-$. 

**Discussion:**

i. We read the features of $f$ from the local graph near $-\infty$ and write the corresponding features of the global input-output rule:

<table>
<thead>
<tr>
<th>Features of the Local Graph near $-\infty$</th>
<th>Features of the Global Input-Output Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>$large \xrightarrow{f} f(\text{large}) = large$</td>
<td>The exponent must be $+$</td>
</tr>
<tr>
<td>$- \xrightarrow{f} f(-) = -$</td>
<td>Either</td>
</tr>
<tr>
<td></td>
<td>the coefficient is $-$ and the exponent is even</td>
</tr>
<tr>
<td></td>
<td>or</td>
</tr>
<tr>
<td></td>
<td>the coefficient is $+$ and the exponent is odd</td>
</tr>
</tbody>
</table>

The global input-output rule of $f$ must therefore be

$$x \xrightarrow{f} f(x) = (-\text{bounded})x^{+\text{even}}$$

or

$$x \xrightarrow{f} f(x) = (+\text{bounded})x^{+\text{odd}}$$

ii. To localize near $0^-$, we compute the output for inputs that are $-\text{small}$.

- If the Input-Output Rule is of the form $x \xrightarrow{f} f(x) = (-\text{bounded})x^{+\text{even}}$
\[-\text{small} \xrightarrow{f} f(-\text{small}) = (-\text{bounded})(-\text{small})^{\text{even}}\]
\[= (-\text{bounded}) \cdot (-\text{small}) \cdot \ldots \cdot (-\text{small}) \quad \text{even number of copies of } -\text{small}\]
\[= -\text{small}\]

- If the Input-Output Rule is of the form \(x \xrightarrow{f} f(x) = (+\text{bounded})x^{\text{odd}}\)

\[-\text{small} \xrightarrow{f} f(-\text{small}) = (+\text{bounded})(-\text{small})^{\text{odd}}\]
\[= (+\text{bounded}) \cdot (-\text{small}) \cdot \ldots \cdot (-\text{small}) \quad \text{odd number of copies of } -\text{small}\]
\[= -\text{small}\]

So, either way, we have that \(-\text{small} \xrightarrow{f} f(-\text{small}) = -\text{small}\)

\(\text{iii.} \) And the local graph near \(0^-\) is:

**I-15.** Let \(f\) be a *power* function whose local graph near \(0^+\) is

Find the local graph of \(f\) near \(+\infty\).

**Discussion:**

i. We read the features of \(f\) from the local graph near \(0^+\) and write the corresponding features of the global input-output rule:
Features of the Local Graph near $0^+$ | Features of the Global Input-Output Rule

| $small \xrightarrow{f} f(small) = large$ | The exponent must be $-$  
| $+ \xrightarrow{f} f(+) = +$ | The coefficient is $+$  
| $- \xrightarrow{f} f(-) = ?$ | We cannot tell the parity of the exponent. |

The global input-output rule of $f$ must therefore be:

either of the form

$$x \xrightarrow{f} f(x) = (+ \text{bounded})x^{-\text{even}}$$

or of the form

$$x \xrightarrow{f} f(x) = (+ \text{bounded})x^{-\text{odd}}$$

**ii.** To localize near $+\infty$, we compute the output for inputs that are $+large$

- If the Input-Output Rule is of the form $x \xrightarrow{f} f(x) = (+ \text{bounded})x^{-\text{even}}$

  $$+large \xrightarrow{f} f(+large) = (+ \text{bounded})(+large)^{-\text{even}}$$

  $$= +\text{bounded}$$

  $$= (+large) \cdot \ldots \cdot (+large)$$

    even number of copies of $+large$

  $$= +small$$

- If the Input-Output Rule is of the form $x \xrightarrow{f} f(x) = (+ \text{bounded})x^{-\text{odd}}$

  $$+large \xrightarrow{f} f(+large) = (+ \text{bounded})(+large)^{-\text{odd}}$$

  $$= +\text{bounded}$$

  $$= (+large) \cdot \ldots \cdot (+large)$$

    odd number of copies of $+large$

  $$= +small$$

So, either way, we have that $+large \xrightarrow{f} f(+large) = +small$
iii. And the local graph near $+\infty$ is:

I-16. Let $f$ be a power function whose local graph near $+\infty$ is

Find the local graph of $f$ near $0^{-}$.

Discussion:

i. We read the features of $f$ from the local graph near $+\infty$ and write the corresponding features of the global input-output rule:

<table>
<thead>
<tr>
<th>Features of the Local Graph near $+\infty$</th>
<th>Features of the Global Input-Output Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>large $\to f(\text{large}) = \text{small}$</td>
<td>The exponent must be $-$</td>
</tr>
<tr>
<td>$+ f \to f(+) = -$</td>
<td>The coefficient must be $-$</td>
</tr>
<tr>
<td>$- f \to f(-) =$?</td>
<td>We cannot tell the parity of the exponent.</td>
</tr>
</tbody>
</table>

The global input-output rule of $f$ must therefore be

$$x \xrightarrow{f} f(x) = (-\text{bounded})x^{-\text{even}}$$

or

$$x \xrightarrow{f} f(x) = (-\text{bounded})x^{-\text{odd}}$$

ii. To localize near $0^{-}$, we compute the output for inputs that are $-\text{small}$

- If the Input-Output Rule is of the form $x \xrightarrow{f} f(x) = (-\text{bounded})x^{-\text{even}}$
\[
-\text{small} \xrightarrow{f} f(-\text{small}) = (-\text{bounded})(-\text{small})^{-\text{even}} \\
= -\text{bounded} \\
= \frac{(-\text{small}) \cdot \ldots \cdot (-\text{small})}{\text{even number of copies of } -\text{small}} \\
= -\text{large}
\]

- If the Input-Output Rule is of the form \( x \xrightarrow{f} f(x) = (-\text{bounded})x^{-\text{odd}} \)

\[
-\text{small} \xrightarrow{f} f(-\text{small}) = (-\text{bounded})(-\text{small})^{-\text{odd}} \\
= -\text{bounded} \\
= \frac{(-\text{small}) \cdot \ldots \cdot (-\text{small})}{\text{odd number of copies of } -\text{small}} \\
= +\text{large}
\]

So, depending on the parity of the exponent, we have that:

either

\(-\text{small} \xrightarrow{f} -\text{large}\)

or

\(-\text{small} \xrightarrow{f} +\text{large}\)

iii. And, depending on the parity of the exponent, the local graph near 0\(^-\)

is either:

or

I-17. Let \( f \) be a power function whose local graph near 0\(^+\) is
Find the local graph of \( f \) near \(-\infty\)

**Discussion:**

i. We read the features of \( f \) from the local graph near \( 0^+ \) and write the corresponding features of the global input-output rule:

<table>
<thead>
<tr>
<th>Features of the Local Graph near ( 0^+ )</th>
<th>Features of the Global Input-Output Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>small ( \rightarrow ) ( f(\text{small}) = \text{small} )</td>
<td>The exponent must be +</td>
</tr>
<tr>
<td>+ ( \rightarrow ) ( f(+) = + )</td>
<td>The coefficient must be +</td>
</tr>
<tr>
<td>- ( \rightarrow ) ( f(-) = ? )</td>
<td>We cannot tell the parity of the exponent.</td>
</tr>
</tbody>
</table>

The global input-output rule of \( f \) must therefore be

\[
x \underset{\rightarrow}{\rightarrow} f(x) = (+\text{bounded})x^{+\text{even}}
\]

or

\[
x \underset{\rightarrow}{\rightarrow} f(x) = (+\text{bounded})x^{+\text{odd}}
\]

ii. To localize near \(-\infty\), we compute the output for inputs that are \(-\text{large}\):

- If the Input-Output Rule is of the form \( x \underset{\rightarrow}{\rightarrow} f(x) = (+\text{bounded})x^{+\text{even}} \)

\[
-\text{large} \underset{\rightarrow}{\rightarrow} f(-\text{large}) = (+\text{bounded})(-\text{large})^{+\text{even}} = (+\text{bounded}) \cdot (\underbrace{-\text{large} \cdot \ldots \cdot -\text{large}}_{\text{even number of copies of } -\text{large}}) = +\text{large}
\]

- If the Input-Output Rule is of the form \( x \underset{\rightarrow}{\rightarrow} f(x) = (+\text{bounded})x^{+\text{odd}} \)
So, depending on the parity of the exponent, we have that:

either 
\[-\text{large} \xrightarrow{f} +\text{large}\]

or 
\[-\text{large} \xrightarrow{f} -\text{large}\]

iii. And, depending on the parity of the exponent, the local graph near 
\(-\infty\) is either:

\[\text{I-18. Let } f \text{ be the function specified by the global input-output rule } x \xrightarrow{f} f(x) = (+29.73)x^1\]

Find the local graph of \( f \) near \( 0 \).

**Discussion:** Since the exponent is \(+1\), the function \( f \) is one of the two kinds of **exceptional** power functions, namely \( f \) is a **dilation** and the graph of \( f \) is a **straight line**.

To get the local graph of \( f \) near 0,

i. We **normalize** the global input-output rule , 
\[x \xrightarrow{f} f(x) = (+\text{bounded})x^1\]
\[= (+\text{bounded})x\]

ii. To **localize** near \( 0 \), we compute the output for inputs that are \( \pm \text{small} \)

\[+\text{small} \xrightarrow{f} f(+\text{small}) = (+\text{bounded})(+\text{small})\]
\[= +\text{small}\]
\[ -\text{small} \xrightarrow{f} f(\text{small}) = (+\text{bounded})(-\text{small}) = -\text{small} \]

iii. So, the place is

iv. and the local graph is

I-19. Let \( f \) be the function specified by the global input-output rule

\[ x \xrightarrow{f} f(x) = (+30.96)x^{-1} \]

Find the local graph of \( f \) near \( \infty \)

**Discussion:** Since the exponent is \(-1\), the function \( f \) is a regular power function.

To get the local graph of \( f \) near \( \infty \),

i. We **normalize** the global input-output rule,

\[ x \xrightarrow{f} f(x) = (+\text{bounded})x^{-1} = \frac{+\text{bounded}}{x} \]

ii. To **localize** near \( \infty \), we compute the output for inputs that are \( \pm \text{large} \)

\[ +\text{large} \xrightarrow{f} f(+\text{large}) = \frac{+\text{bounded}}{+\text{large}} = +\text{small} \]

\[ -\text{large} \xrightarrow{f} f(-\text{large}) = \frac{+\text{bounded}}{-\text{large}} = -\text{small} \]
iii. So, the place is

iv. and the local graph is

I-20. Let \( f \) be a \textit{power} function whose local graph near 0 is

Which of the following, if any, \textit{must} be features of the global input-output rule that specifies \( f \)

- M The exponent must be positive
- N The exponent must be negative
- P The exponent must be even
- Q The exponent must be odd
- R The coefficient must be positive
- S The coefficient must be negative
- T Cannot be

**Discussion:** We read the features of \( f \) from the local graph near 0 and write the corresponding features of the global input-output rule:

<table>
<thead>
<tr>
<th>Features of the Local Graph near 0</th>
<th>Features of the Global Input-Output Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f \rightarrow f(\text{small}) = \text{large} ) ( f \rightarrow f(+) = - ) ( f \rightarrow f(-) = - )</td>
<td>The exponent must be – The coefficient must be – The exponent must be even.</td>
</tr>
</tbody>
</table>

I-21. Let \( f \) be a \textit{power} function whose local graph near \( \infty \) is
Which of the following, if any, must be features of the global input-output rule that specifies $f$:

- M The exponent must be positive
- N The exponent must be negative
- P The exponent must be even
- Q The exponent must be odd
- R The coefficient must be positive
- S The coefficient must be negative
- T Cannot be

**Discussion:** We read the features of $f$ from the local graph near $\infty$ and write the corresponding features of the global input-output rule:

<table>
<thead>
<tr>
<th>Features of the Local Graph near $\infty$</th>
<th>Features of the Global Input-Output Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{large} \xrightarrow{f} f(\text{large}) = \text{large}$</td>
<td>The exponent must be $+$</td>
</tr>
<tr>
<td>$+ \xrightarrow{f} f(+) = +$</td>
<td>The coefficient must be $+$</td>
</tr>
<tr>
<td>$- \xrightarrow{f} f(-) = +$</td>
<td>The exponent must be even.</td>
</tr>
</tbody>
</table>

**I-22.** Let $f$ be a power function such that $\text{Height-size} f|_{\text{near } \infty} = (\text{small}, \text{small})$.

Which of the following, if any, must be features of the global input-output rule that specifies $f$:

- M The exponent must be positive
- N The exponent must be negative
- P The exponent must be even
- Q The exponent must be odd
- R The coefficient must be positive
- S The coefficient must be negative
- T Cannot be

**Discussion:** We read the information given about $f$ and write the corresponding features of the global input-output rule:

<table>
<thead>
<tr>
<th>Information given about $f$</th>
<th>Features of the Global Input-Output Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+\text{large} \xrightarrow{f} f(+\text{large}) = \text{small}$</td>
<td>The exponent must be $-$</td>
</tr>
<tr>
<td>$-\text{large} \xrightarrow{f} f(-\text{large}) = \text{small}$</td>
<td>The exponent must be $-$</td>
</tr>
</tbody>
</table>

Since no information is given about $\text{Sign}$, nothing can be said about the sign of the coefficient and therefore nothing about the parity of the exponent.
I-23. Let $f$ be a power function such that $\text{Height-sgn}\ f_{\nearrow}\infty = (+,-).$

Which of the following, if any, must be features of the global input-output rule that specifies $f$

M The exponent must be positive     N The exponent must be negative
P The exponent must be even         Q The exponent must be odd
R The coefficient must be positive   S The coefficient must be negative
T Cannot be

Discussion: We read the information given about $f$ and write the corresponding features of the global input-output rule:

<table>
<thead>
<tr>
<th>Information given about $f$</th>
<th>Features of the Global Input-Output Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+\text{large} \rightarrow f(+\text{large}) = +$</td>
<td>The coefficient must be $+$</td>
</tr>
<tr>
<td>$-\text{large} \rightarrow f(-\text{large}) = -$</td>
<td>The exponent must be odd</td>
</tr>
<tr>
<td>No information given about Size $f(x)$</td>
<td>We cannot tell the sign of the exponent.</td>
</tr>
</tbody>
</table>

If we have the graphs of the eight types of regular power functions in mind, there is another way to proceed namely by seeing that there are only two types for which $\text{Height-sgn}\ f_{\nearrow}\infty = (+,-)$. More precisely, $\text{Height-sgn}\ f_{\nearrow}\infty = (+,-)$ gives: which gives the place:

which shows that there are only two possible local graphs:

or
We can then see that

- The coefficient must be $+$
- The exponent must be odd

but

- The exponent could be either $+$ or $-$

**I-24.** Let $f$ be a regular power function such that $\text{Slope-sign} f|_{near \ 0} = (\swarrow, \searrow)$.

Which of the following, if any, must be features of the global input-output rule that specifies $f$

- M The exponent must be positive
- N The exponent must be negative
- P The exponent must be even
- Q The exponent must be odd
- R The coefficient must be positive
- S The coefficient must be negative
- T Cannot be

**Discussion:** Since the given information is about the Slope-sign, we cannot get the features of the global input-output rule straight from the given information and we absolutely need to have the graphs of the eight regular power functions in mind.

But then we can see that there are only two regular power functions for which $\text{Slope-sign} f|_{near \ 0} = (\swarrow, \searrow)$:

![Graph](image)

from which we see that

- The exponent must be odd

but

- The coefficient could be either $-$ or $+$
- The exponent could be either $+$ or $-$

**I-25.** Let $f$ be a power function such that $\text{Concavity-sign} f|_{near \ 0} = (\cap, \cup)$

Which of the following, if any, must be features of the global input-output rule that specifies $f$
M The exponent must be positive  \hspace{1cm} \text{N The exponent must be negative} \\
P The exponent must be even  \hspace{1cm} \text{Q The exponent must be odd} \\
R The coefficient must be positive  \hspace{1cm} \text{S The coefficient must be negative} \\
T Cannot be \\

\textbf{Discussion:} Since the given information is about the \textit{Concavity-sign}, we cannot get the features of the global input-output rule straight from the given information and we absolutely need to have the graphs of the eight regular power functions in mind.

But then we can see that there are only two regular power functions for which Concavity-sign $f|_{\text{near } 0} = (\cap, \cup)$:

\begin{itemize}
  \item The exponent must be \textit{odd}.
  \item The coefficient must be $+$
\end{itemize}

but

\begin{itemize}
  \item The exponent could be either $+$ or $-$
\end{itemize}