A Reasonable Sequence? For Real? II.

July 14th, 2009

Summary of Past Episode: After some forty years of not so benign neglect, and under somewhat clumsy pressure on the part of the Vice-President For Academic Affairs, the mathematics department, suddenly become aware that the immense majority of its students were in the developmental courses, appointed a committee to deal with the matter. Two or three years later, the state of developmental students had not improved one bit but it looked as if my own students who are using **Reasonable Basic Algebra** might be threatened by the Final Exams to come. And then, quite by accident, I happened to have another conversation with the "presumably sick and tired administrator" at my school whom I mentioned earlier in **Memorandum For A Reasonable Sequence** during which we discussed the shape of the learning curve in **RBA**.

I had of course been criticizing the output of CEMEC, the committee rather unfairly empowered by the department to solve the problem while business could continue as usual, as if nothing was remiss. While they are nowhere near what commercial textbooks are now down to, the texts CEMEC had developed were nevertheless not much more than collections of topics even if written in reaction to carefully catalogued "elementary" errors committed by students in subsequent courses.

The question, though, is whether merely saying "this is a mistake and here is the correct way to do it" has any chance of being successful. The telltale is the huge number of exercises even if not repetitive and I cannot see that this is anything but memory-based. My own response, expounded in Chapter Thirteen, is that not making such mistakes can only result from the students understanding what they are doing and the necessity of communicating what they were doing to others without ambiguity: "No, I don't know what you mean".

The question the "presumably sick and tired administrator" then asked was of course how *my* students were doing and I had to admit that I didn't really know. My excuse was, equally of course, that one semester was too short really to have a measurable impact. The administrator, though, persisted: what was my feeling? I then explained that my students essentially go through three phases.

1. During a first phase, which roughly coincides with **Part One – Elements of Arithmetic**, students flatly refuse to believe the characterization of mathematics as something in which one is more interested in trying to make sense of what is going on rather than "just learning how to do" a bunch of items to be on the exam. As a result, they try to recall, more or less successfully, whatever they remember of Arithmetic.

However, with questions such as

On Monday, your balance was two hundred thirty four dollars and fifty six cents in the red. On Tuesday, you deposit sixty eight dollars and eighty three cents. What is your new balance?

which require to be read, the students are not too successful even though they have been aware of the kind of questions all along. They thus tend to be angry.

2. During a second phase, which roughly coincides with **Part Two – Problems**, the students start taking the stuff a bit more seriously, in part probably because the stuff is not so "familiar" but the level of anxiety rises correspondingly because they keep making errors in problems such as

   Find the solution subset of the double problem:

   either $x<+21.46$ or $x\geq-53.03$ but not both.

   while realizing now that the problems are not difficult.
3. However, in the third phase, which roughly coincides with **Part Three – Laurent Polynomials**, students begin to accept the fact that they really *could* "do all the work" without me telling them what to do but now they see that time is now running out. Nevertheless, and perhaps surprisingly, they are not angry anymore.

So, what I was of course deploring was that the students in the next semester will be back to square one and likely disgruntled while there was really no reason for not building on this change of attitude.

And then, it struck me: since the department had cut down polynomials from the *Basic Algebra course*, **Part Three – Laurent Polynomials,** could become Part One of a **Reasonable Intermediate Algebra (RIA)**. This would also have the advantage that **Part One – Elements of Arithmetic**, in addition to the introduction of the notion of function already advertised, could be developed a bit to include decimal numbers and the notion of approximation.

In view of the fact that nothing more really was needed for **Reasonable Algebraic Functions**, the way to **Differential Calculus** was paved.

And then, it seemed as if not only ought the *Basic Algebra* course be linked with an *English Reading* oourse, but that the *Intermediate Algebra* course could be similarly linked with an *English Writing* course in which students would learn how to write their cases on mathematical problems. The argument was that just as with reading mathematics, writing mathematics ought to transfer rather easily to writing "convincing arguments".

The question remained, though, as to what the rest of **RIA** ought to consist of. The "presumably sick and tired administrator" let me have the course description for the *Intermediate Algebra* course which I annotated as follows:

A critical reading by Schremmer of the 1991 course description of **Intermediate Algebra**.

I. Real Number System (CLO I)
A. Identify real numbers, natural numbers, integers, rational numbers and irrational numbers
B. Determine the order of real numbers
C. Identify some of the properties of real numbers

Notes.
This is sheer make believe. How can definitions be provided in the absence of a serious conceptual grasp of equations. As a matter of sheer nomenclature to be memorized, it is not very desirable as a beginning. For example, how is one to compare $\sqrt{15}$ and π in current *Intermediate Algebra*?

In alternate *Basic Algebra*, RBA chapters 1 to 6 as to be modified plus two chapters on decimal numbers and decimal approximations, things make “perfect sense”as decimal approximations of solutions of equations:

From $x^2 = 15$, we get with the help of $(x_0+h)^2 = x_0^2 + 2x_0 \cdot h + h^2$ (RBA chapter 17) that the approximate solution—called $\sqrt{15}$—is, depending on the level of approximation
$\sqrt{15} = 3 +$ [...]
$\sqrt{15} = 3.8 +$ [...] or $3.9 +$ [...]
and given that $\pi = 3.1 +$ [...]
we have that π is less than $\sqrt{15}$

II. First Degree Equations and Inequalities (CLO II)
A. Solve equations including fractional forms
B. Solve literal equations
C. Solve inequalities
D. Solve absolute value equations and inequalities
E. Solve word problems using first degree
equations and inequalities

Note. At best, this is an ill-defined grab bag.
A, B and C are done properly in RBA chapters 7 to 11.
D is not done in **RBA** nor will it done in the alternate *Intermediate Algebra* for two reasons. **RBA** does discuss the fact that one can compare signed numbers from either the signed point of view or the point of view of “size” but at no

point in the development does there arise a need for a systematic investigation of (in)equations involving sizes. The other reason is that although it could be easily done on the basis of what is already there, it seems more reasonable to allocate what little time there is to what cannot be dispensed with.
E was very fashionable in the nineties but has started to be discredited by somewhat serious research.

III. Polynomials (CLO III)
A. Determine the degree of a polynomial
B. Classify a polynomial by number of terms
C. Perform factorization of polynomials including multiple step factoring
D. Translate word problems into equivalent quadratic expressions
E. Solve equations of the form (A) (B) = O where A and B are polynomials
Notes.
A and B are both jokes. Students have no problem with degree and number of terms once they have familiarized themselves with polynomials.
C is downright criminal inasmuch as there is NO procedure to factor polynomials even of degree 2.
D. See above
E. This is nothing more than the fact that if you know that two numbers multiply to 0, then you know that one at least has to be 0. This has nothing to do with algebra.

IV. Algebraic Fractions (CLO IV)
A. Reduce algebraic fractions
B. Multiply and divide algebraic fractions
C. Add and subtract algebraic fractions
D. Simplify complex algebraic fractions
E. Reduce algebraic fractions after adding or subtracting
F. Solve word problems using algebraic fractions
G. Solve literal equations with algebraic fractions
Notes.
A to E make sense only once the relevant arithmetic ideas have been understood. And then, the transfer to algebra poses little difficulty. Moreover, while there is really no need for algebraic fractions by themselves, they do not pose any difficulty in the investigation of Rational Functions.

F and G. See above
LAST BUT NOT LEAST: It is difficult to see how “addition, subtraction, multiplication, division—both in ascending and descending order of exponents—of Laurent polynomials, (that is including negative exponents) together with the expansion of (x_0+h)^n (binomial theorem)” can be neither in “redefined” *Basic Algebra* nor in current *Intermediate Algebra* !!!!

V. Exponents and Radicals (CLO V)
A. Simplify expressions with rational exponents
B. Simplify radical expressions
C. Add, subtract, multiply and divide radical expressions
D. Solve equations containing one or more radical expressions
E. Solve word problems using radical expressions
Notes,
A. Rational exponents is a very sophisticated notation for which there is no use at this level: For instance, 17^(3/5) is the name of the solution of the equation x^5 = 17^3, not exactly very likely to be encountered at this level.
B and C are quite surprising both in view of the absence of their counterpart for “polynomial” expressions and of the lack of context here.
D is beyond the pale
E See above.

VI. Second Degree Equations and Inequalities (CLO VI)
A. Solve second degree equations
B. Solve second degree literal equations
C. Solve second degree inequalities
D. Solve word problems using second degree equations and inequalities

Notes. This is more than a bit glib as the obstruction to solving second degree equations is that there is one more term than the = sign has sides so that it is a far cry from solving affine equations. In any case, they are best left out of *Intermediate Algebra*.for the same two reasons as in CLO II: There is no need for the investigation or quadratic equations until quadratic functions are investigated—which is where everything makes perfect sense—and the time is better employed elsewhere. This properly belongs to *Precalculus One*.

VII. Rectangular Coordinate System (CLO VII)
A. Determine the slope and intercepts of a line
B. Determine the equation of a line
C. Determine parallel and perpendicular lines
D. Graph linear equations
E. Graph parabolas
Notes
A is misleadingly worded: The slope of a line is most important. The y-intercept is the output for input 0 and thus of little interest. The x-intercept on the other hand is the input for which the output is 0 and therefore the solution of an affine equation.
B usually consists of:
—the “point-slope formula” which is better seen as a prototypical “Initial Value Problem” (IVP) in the case of affine functions. This properly belongs to *Precalculus One*.
—the “two-points formula” which is better seen as a prototypical “Boundary Value Problem” (BVP) in the case of affine functions. This properly belongs to *Precalculus One*.
Because both types of problems are extremely important—but also extremely difficult type of problems—it is certainly worth looking at these prototypes but this requires the context of affine functions.
C is of secondary importance at this stage and should be omitted from alternate *Intermediate Algebra* for the usual two reasons.
D is out of context as “graphing linear equations”—presumably in two variables—belongs to 2-dimensional geometry and should be omitted from alternate *Intermediate Algebra* for the usual two reasons.. This is in contradistinction with the graphing of affine functions.
E. This makes sense only once parabolas have been defined which they don’t seen to have been here. This properly belongs to *Precalculus One*.

VIII. Relations and Functions (CLO VIII)
A. Define relations and functions
B. Evaluate a function
C. Graph relations and functions
D. Translate word problems into equivalent functional expressions
E. Solve word problems using functional notation

Notes
A, B. This makes little sense at this late stage inasmuch as it cannot lead anywhere here. See Epilogue in **RBA**.
C. This is completely misleading inasmuch as it uses the word "graph" instead of the word "plot". What happens of course, is that other than with affine functions, there is absolutely no way to turn a plot into a graph by just "joining smoothly" the plot points. A lot of qualitative information must be dug out of the input-ouput rule of the function before this can be done. This is in fact the theme of **RAF**, the text I developed for *Precalculus One*.
D. See above
E All mathematicians fervently wish this were possible.

IX. Systems of Equations (CLO IX)
A. Solve systems of linear equations in two variables:
1. Using elimination
2. Using substitution
B. Solve word problems using systems of linear equations in two variables
C. Solve nonlinear systems in two variables
Notes
A comes rather late since "solving systems of [presumably two] linear equations in two variables" is a necessity for VII-B. In fact it is dealt with in **RBA** chapter 12.
B See above
C is a howler in vagueness. Besides, it makes no sense until functions beyond affine functions have been introduced. This properly belongs to *Precalculus One*—if at all. Should be omitted from alternate *Intermediate Algebra* for the usual two reasons.

So, on prima facie evidence, it seemed that it should not be too hard to develop **RIA**

[To be continued]

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