An Adventure In Academia, II

March 29th, 2009

Summary of Past Episodes: We left me rather amazed at the fact that, somehow, someone, somewhere in academia, had actually noticed that the great masses were being served rather substandard mathematics, didn't in fact seem to think that this was something pre-ordained and therefore that one might at least *think* about it and, possibly even *do* something about it.

So, … having been enraptured by such political overture in a call for proposals to discuss issues of "quality versus equity", but also for other reasons to be discussed later, I responded with the following:

Please find below my "*expression of interest*":

An often overlooked aspect of "developmental mathematics education", at least in the USA, is that commercially available textbooks are memory based so that, to facilitate memorization, the subject matter is atomized into "topics" presented independently during a couple of short semesters so that nothing can make any sense anymore. All connective tissues have been removed and no build-up can take place. Indeed, typically, instructors deplore that their students cannot remember the simplest things past the test. This has a number of dire consequences:

1. Students who wish eventually to learn, say, Differential Calculus, the "mathematics of change", face an inordinate number of courses: Arithmetic, Elementary Algebra (8/9th grade Algebra I), Intermediate Algebra (10/11th grade Algebra II), College Algebra, College Trigonometry, Calculus I.

2. Success is defined internally rather than by success in later courses with the result that it is rarely noticed that the chances of overall success are extremely low—in the above example, usually no more than one percent.

3. Developmental students are ghettoized into identifying learning with having recourse to experts and into belittling the power of personal logical thinking while, to quote Colin McGinn, "One of the central aims of education, as a preparation for political democracy, should be to enable people to get on better

terms with reason—to learn to live with the truth."

Yet, there is nothing inevitable about this situation and the object of the proposal is the description of a three course sequence in which the intention is to get the students to change, in John Holt's words in "How Children Fail", from being "answer oriented", the inevitable result of "show and tell, drill and test", to being "question oriented" and thus, rather than to try to remember things, the students can reconstruct them if and when needed.

Some of the characteristics of this sequence are:

—An expositional approach based on what is known in mathematics as Model Theory which carefully distinguishes "real-world" situations from their "paper-world" representations.

—Contents carefully structured into an architecture designed to create systematic reinforcement and thus foster an exponential learning curve based, in Liping Ma's terms, on a "Coherent View of Mathematics" and thus help students acquire a "Profound Understanding of Fundamental Mathematics".

—Systematic attention given to linguistic issues that often prevent students from being able to focus on the mathematical concepts themselves.

—Continuing insistence on convincing the students that the reason the things they are dealing with are the way they are is not because "experts say so" but because common sense says they cannot be otherwise. But, while the standard way of establishing truth in mathematics is by way of proof, Edward Thorndike showed a century ago that proofs do not transfer into convincing arguments. So, the sequence uses a mode of arguing somewhat like that used by lawyers in front of a court. See Toulmin's The Uses of Argument.

Such a "fast track" as described above would of course seem utterly improbable and the reason it works is that the investigation of functions is based on (Laurent) polynomial approximations as an extension of decimal approximations. i.e. Lagrange's approach.

Part Two, Algebraic Functions, and Part Three, Transcendental Functions, were indeed originally specified, designed and experimented with under the terms of a 1988 NSF calculus grant as an alternative to the conventional sequence, Precalculus I, Precalculus II and Calculus I (Differential). The 1992 report of my school's Office of Institutional Research said in part:

"Of those attempting the first course in each sequence, 12.5% finished the [conventional three semester 10 hour] sequence while 48.3% finished the [integrated two semester 8-hour] sequence, revealing a definite association between the [integrated two semester 8 hour] sequence and completion ($\chi 2$ (1) = 82.14, $p < .001$)."

The report also said that the passing rates in Calculus II (Integral) for the students coming from the above two sequences were almost identical but that this was not significant because, in both sequences, most students did not continue into Calculus II (Integral).

Because of the work done for Differential Calculus I and Differential Calculus II, the contents of the Algebraic Functions course and of the Transcendental Functions course are well specified and therefore the contents of Part One, Decimal Arithmetic and Basic Algebra are fairly well specified as being exactly, no more, no less, what is needed for the Algebraic Functions and Transcendental Functions courses. They are currently under development.

Then, upon a little bit of reflection in which I thought that the above sounded a bit too much like self-promotion, I emailed the following:

Being at the end of my career, I am not in need of publications nor do I even particularly wish to publish as the situation described in the first half of the rational I submitted concerns me enough that I got myself involved in the rather large project alluded to in the second half of the rational. See freemathtexts.org for more detail.

On the other hand, the alternative given in the call for proposals, i.e. "[q]uality mathematics [...] seen as a reflection of its rigor, formality and generalisability" versus "utilitarian importance" struck me as rather restrictive and I would argue that there is a middle ground in which the situation mentioned in the first half of my proposal is dealt on the basis of an appeal to "reason" as opposed to a reliance on memory and which addresses utilitarian needs much better than the passive ingurgitation of "recipes" as only sketched in the second half of my proposal.

As it happens, I have just begun reading Tao's "Why are solitons stable" in the current Bulletin and I was struck by the way Tao was dealing with the subject and the way he was talking about it: Neither the usual, "Let f …" nor the degrading style found in today's textbooks which take their readers for utter idiots.

So, it occurred to me that I should have perhaps proposed a different light than I did and put the emphasis on the "third road", that is one in which the mathematical exposition is neither "rigourous" in a pseudo Bourbaki manner—and, essentially, unreadable—nor down-graded into utilitarian cookbooks—with built-in obsolescence—and is accessible to the "great unwashed masses". So, I thought that I should argue that a logical mind is an asset, not only in many practical situations (I still do general construction work in the summer) but also in societal matters as noted above in the quote from Colin McGinn and what is of paramount importance is that mathematics can be the simplest environment in which to develop one's mind. It is not "problem solving" that is important, it is that a logical mind is what allows us to deal with many a priori different problems, recognize that, after all, they have similarities and learn from the one to deal with the other. Of course, after he got the Field medal, that this is the way that Tao works has been criticized, for not being specialized.

So, rather than to present an actual "solution" (mine), I should perhaps discuss the necessary parameters of any solution (using my own, easily accessible, solution for examples), in particular the time necessary for the learning curve to become exponential … but also for the conditions necessary for other, not a priori convinced people to give it a try:

"Early in my career, I naively thought that if you give a good idea to competent mathematicians or physicists, they will work out its implications for themselves. I have learned since that most of them need the implications spelled out in utter detail."

–Hestenes, Oersted Lecture, page 38

Now, while I have very little respect for, and patience with, referees and all because they always want you to have written the piece that they would have wanted you to write, and because I think that editors, by contrast, at least know who the readership will be, I would like to ask you which proposal would be more in line with the goals of the volume.

In any case, I hope you will forgive me for this overlong afterthought.

Thereupon, on the response that “[the thoughts] are definitely worth expanding”, I sent in the following:

Mathematics education has been confronting the problem of how to bring mathematics to the “great unwashed masses” for at least thirty years but with no discernible success or even progress. In fact, the only conspicuous thing is that mathematics textbooks during that time have devolved to exposition by way of “template examples” and that the subject matter has been atomized into “topics” presented independently to facilitate memorization while, typically, instructors deplore that their students cannot remember the simplest things past the test.

Of course, it is not difficult to show how the stress generated by memorization on the scale required by, say, a year of mathematics must necessarily have that result. However, the operating, if tacit, assumption is that “developmental” students are incapable of learning on the basis of logic, the only alternative to memorization. And by an unfortunate, even if possibly unavoidable, coincidence, not only has research in mathematical learning also largely dealt with isolated topics but, even more unfortunately, it too has essentially equated learning with memorizing.

I would argue that the alternative in the call for proposals, i.e. “[q]uality mathematics [...] seen as a reflection of its rigor, formality and generalisability” versus “utilitarian importance” would seem to be rather beside the point given that there is a third approach in which, i. learning is done on the basis of an appeal to “reason” as opposed to a reliance on memory and, ii. utilitarian needs are much better addressed than with the passive ingurgitation of “recipes” whose obsolescence is built-in in that a logical mind is very much an asset in practical situations not to mention that “One of the central aims of education, as a preparation for political democracy, should be to enable people to get on better terms with reason—to learn to live with the truth.” (Colin McGinn).

The first thing that this third approach requires is of course what Liping Ma called a “coherent view of mathematics” which, in turn requires a carefully designed contents architecture in which students cope with progressively more complicated situations and, even more importantly, in which one thing leads naturally to another and which gives students time to reflect.

While the recourse to “concrete applications” has finally been shown to be rather counter-productive, this is not to say that mathematics should not derive from the real world. In fact, what is quite natural is an expositional approach based on what is known in mathematics as Model Theory which carefully distinguishes “real-world” situations from their “paper-world” representations.

The third thing that has to be carefully dealt with is the meta-language, that is the language used to present and discuss the object language used by mathematics to represent the real-world. Systematic attention has to be given to linguistic issues that often prevent students from being able to focus on the mathematical concepts themselves.

Last, and most importantly, the students must become convinced that the reason the things they are dealing with are the way they are is not because “experts say so” but because common sense says they cannot be otherwise. But, while the standard way of establishing truth in mathematics is by way of proof, Edward Thorndike showed a century ago that proofs do not transfer into convincing arguments. So, a mode of arguing more like that used by lawyers in front of a court is necessary. See Toulmin’s The Uses of Argument.

Quite obviously, this cannot be done in the course of a couple of semesters but, perhaps surprisingly, there is strong evidence that it can be done in three four-hour semesters.

I would propose to discuss the above in some depth and to offer, as proof of concept, work done under the terms of a 1988 NSF calculus grant as an alternative to the conventional sequence, Precalculus I, Precalculus II and Calculus I (Differential). The 1992 report of my school's Office of Institutional Research read in part:

"Of those attempting the fi rst course in each sequence, 12.5% finished the [conventional three semester 10 hour] sequence while 48.3% finished the [integrated two semester 8-hour] sequence, revealing a definite association between the [integrated two semester 8 hour] sequence and completion (?2 (1) = 82.14, p < .001)."

The report also said that the passing rates in Calculus II (Integral) for the students coming from the above two sequences were almost identical but that this was not significant because, in both sequences, most students did not continue into Calculus II (Integral).

What is directly relevant to "developmental" mathematics is that what made the above sequence work is the systematic use of (Laurent) polynomial approximations (Lagrange's approach) and that these are of course nothing but an extension of decimal approximations so that a "profound understanding of fundamental mathematics", in this case functions, decimal approximations, equations and inequations, and (Laurent) polynomials, is all that is necessary and is likely achievable in one four-hour semester. Given the pass rate mentioned above, an overall success rate from Arithmetic to Differential Calculus ought, no matter what, to be considerably higher that the current one, usually no more than one percent. See freemathtexts.org

But then common sense, you might say, prevailed and I forgot all about it.

[To be continued, though.]

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