# A Reasonable Sequence? For Real? I.

May 29th, 2009

It would seem that I may not have been entirely fair to the "presumably sick and tired administrator" at my school whom I mentioned in **Memorandum For A Reasonable Sequence**. Things seem to be perhaps moving but, at best, with very, very deliberate speed and what is going to happen remains totally unclear.

But first let me go back and provide a bit of background even though the story is rather unlikely to be unimaginable to people in the profession.

A few years ago, the Vice President for Academic Affairs took notice of the fact that the Mathematics Department hadn't exactly gone out of its way to facilitate access to mathematics for the "great unwashed masses". We do have the required Arithmetic and Basic Algebra non-credit courses as well as the usual Intermediate Algebra credit course from which students are supposed to be able to get into PreCalculus One and then PreCalculus Two and then, if they are still alive, Calculus One—Differential. (There are of course other "Paths To Possibilities" but this is the one I am interested in.) What the VP may have noticed was that precious few students make it through that path: Less than a quarter of one percent of the students registering in Arithmetic pass Differential Calculus. What the VP actually complained about, though, was the passing rates in the first two courses of the "path": Arithmetic and Basic Algebra.

To remedy this unfortunate situation, the VP—who is nowhere near being a mathematician—thought to order the Department to hire people with a degree in Math Ed instead of in just Mathematics. The underlying reasoning, though, was not clear since a couple of "Educologists" could not possibly make any difference given the very large number of developmental sections to be taught. Still, that would be good for the moral of the few "educators" in the Department whom the VP is said to approve of. You know the kind.

After the Department tried to stonewall the VP by explaining that it was the students who were to blame, or maybe the students' parents, or the high schools, or the use of calculators, or the lack of time or maybe El Niño, or who knows what, the VP thought of applying some pressure on the Department by threatening to create a separate Developmental Mathematics Department.

The pressure was not all the VP could have wished because, if the VP could probably have forced the Chairperson of this Developmental Mathematics Department to be an "Educologist", since there is a Collective Bargaining Agreement most of the current mathematics faculty, including yours truly, would have ended up teaching in the Developmental Mathematics Department.

Still, the fear in the Department was such that it bestirred itself to the point that . . . it created a new **Committee of Elementary Education and its Effects on the Curriculum** (CEMEC) which, sure enough, eventually came up with a **Proposal**.

The Proposal invoked all the appropriate "authorities", the "Crossroad in Mathematics Standards", etc, and said that it had

examined our curriculum and the way it is taught and concluded that:

It is quite often presented in a very repetitive manner. A teacher presents a problem and the same type of problem, where the variation does not explore the complexities inherent in the recursive scheme of the algorithm, is solved by students several times, with no additional gain in insight beyond the trivialities of the variation.

The textbooks that are commonly used (and, in some way, adopted by the Department) reinforce that type of teaching.

The conclusion was lofty enough but the implementation was essentially to be fewer topics—which was good but one might perhaps be forgiven for wondering what took the Department so long to come to that conclusion—and increasing the number of instruction hours—which was debatable and most certainly a tactical mistake. And, last but not least, texts were of course to be written.

The "area of attention" was confined to: "Teaching issues, Faculty preparation and responsibiliites, Students' issues, Structure of remedial mathematics program." In other words, rather ironically, exactly what you would have expected from . . . Educologists!

What was glaringly absent was any"attention" to what was to be *learned* by the students. The 116 page long document did "specify" the courses with detailed lists of topics, each illustrated with sample questions but the emphasis throughout remained on mathematical "factoïds" and the issue of what bound them and why they should be investigated was not even raised. Typical was the use of phrases such as "understanding of concepts"—even though they were to remain in isolation—and "proper use of mathematical language and symbols"—as if in a "finishing school".

For example, in the exercise

Write using exponential notation whenever possible {(-z)(-z)(-z)} / {z+z+z}

students must not only recognize the operations that are performed on the variable z but also recall that exponential notiation applies only to multiplication. Each time a student decides whether exponential notation can be used, he reinforces his understanding of this concept.

And, in fact, the texts that were eventually to be written were entirely prescriptive.

The VP agreed to a "pilot testing" but, a couple of years later, the results were rather unsurprising:

It is clear that pass rates for students in CEMEC sections were no better and in almost all cases worse than for students in the non-experimental sections.

But of course this was because

the CEMEC materials are more demanding and since the students in CEMEC sections were consistently held to higher standards than in most other sections, it is not surprising that pass rates in CEMEC sections are lower than in other sections.

Some attention *was* given to the outcome in further mathematical courses:

The success of students in subsequent mathematics courses was always considered by CEMEC to be a prime indicator of the success of this approach. The results are mixed. The success of Arithmetic students in passing Basic Algebra is roughly the same for students in CEMEC and non-CEMEC sections. The success of Basic Algebra students in passing Intermediate Algebra is clearly better for students in CEMEC sections compared to students in non-CEMEC sections.

However,

The positive reports of faculty in the Pilot provide qualitative evidence for the sustainability and possible expansion of the CEMEC plan. The materials and methods of CEMEC have been embraced by a diverse group of mathematics instructors with different histories, practices and assumptions.

And therefore

The CEMEC Pilot Project has shown promise in accomplishing its primary goal: to have students who successfully complete Arithmetic and Basic Algebra obtain a better understanding of Arithmetic and Algebra. Where it has fallen short is in getting a larger percentage of students to successfully complete these courses.

The VP was not happy, rejected CEMEC's conclusions, demanded Exit Criteria and independently administered Final Exams with, again, the threat of a separate Developmental

Mathematics Department lurking in the background.

The Department caved-in re. Math Ed degrees and approved "Revised" Course Descriptions for Arithmetic and Basic Algebra together with mandatory Common Final Exams.

Regarding the latter, I wrote the following:

I have long been (40+ years) an advocate of common exams for the simple reason that most courses at CCP are part of sequences—whether at CCP or continuing elsewhere—so that the instructor in course n has to know what s/he can count on the students having learned in courses i < n. Moreover, since most courses at CCP are multi-sections, instructors need to tune their violins.

I have long been (50+ years) an anarcho-syndicalist. For those to whom the term is less than familiar, it designates the school of anarchism whose main concern is to prevent the accretion of power in the hand of a minority but which realizes that, while small may be beautiful, certain systems need to be large, e.g. airlines, railroads, manufacture, Basic Algebra, etc. However, anarcho-syndicalism holds that the solution is not management by a small minority but by "all involved" through syndicalism—as partially opposed to unionism in its current meaning.

As such, here is the kind of system that I advocated back then. Let me take an exam for Basic Algebra as an example. Say the exam is to consist of 25 questions.

PART ONE: Specifying the exam. (Or maybe the course?)

1. Let everybody—everybody who wants (?)—submit, say, 10 different questions
2. This should add up to at least a couple of hundred questions.
3. Partition these questions according to roughly what they intend to check, e.g. addition in Z, division of polynomials, etc so that each part corresponds to what I will call, for lack of a better term, a "checkable item".
4. If necessary, subdivide the above parts. For instance, division of polynomials might be divided into: with all coefficients in N, or in Z, or in Q^+ or in Q. Will division in ascending exponents be included? Etc.

5. Work on this until an acceptable list of, say, 30 "checkable items" has been arrived at. This list of checkable items is just an embodiment of a list of descriptions of "intermediate performance objectives" which may or may not actually be written down eventually—although I don't see the point. Hopefully, the fact that the length of the list is bounded (here 30)—although not by the number of questions on the exam (here 25)—should help a consensus to be arrived at. The fact that there are more checkable items than items actually checked on any exam actually given out ought to prevent, at least to some extent, "teaching to the exam".

PART TWO: Implementing the exam.

1. For each one of the 30 "checkable items", let everybody—who wants (?)—submit, say, one "instantiation", that is an actual question.
2. Let everybody see all the instantiations proposed for all the checkable items and let everybody have the right to reject any instantiation(s) but only with an explanation as to why. For example, if I submit an instantiation of "division of polynomials" in which a polynomial with seven terms has to be divided by a polynomial with four terms, one may object that such length is not necessary to check whether the student can really divide a polynomial by another. Else, I would have to explain exactly why such long polynomials are needed. On the other hand, were you to submit 2+3 as an instantiation of addition in N, I would reject it on the basis of various objections.
3. Note: The distinction between "checkable item" and "instantiation" of a checkable item is thus paramount: One might agree with a "checkable item" but not with a "instantiation" proposed for that checkable item.

Given that there are very many sections of Basic Algebra, such a system would ensure that all the students would be taking essentially the same exam while no two sections would have exactly the same version and an instructor might even use two different version in her/his class. This would call for, I think, about a dozen instantiation of each checkable item for the system to be reasonably reliable.

It would also require a computerized system to produce the actual exams. Fortunately, there are several LaTeX packages that can do this. The one I am using is probsoln. I used it to write my own implementation of the system described above. For each instantiation, this implementation allows both a

multiple-choice format and an open format. For instance, given (+3) + (-2) + (-3) + (+5) = ?, this implementation can, for instance, propose the choices (a) +3, (b) –3 . . . (e) None of the preceding. It can also give a "response space"as well as other options. Since I was able to write this implementation, anyone should be able to but, in any case, it will soon be available for free download under a GNU Free Documentation License.
Beyond this issue of common exams, we should also think about the specific "goals" of each sequence and then how to achieve these goals over the length of the sequence as opposed to listing a laundry list of "skills" as nowadays offered in place of "intermediate performance objectives".

Of course, the Department found it easier to delegate the whole thing to a committee and I started seriously to worry about how I would protect my Basic Algebra students—who are using **Reasonable Basic Algebra**—from a final exam likely to focus on "understanding of concepts" and "proper use of mathematical language and symbols".

This was when, somehow, something stirred along the lines of the **Memorandum For A Reasonable Sequence**.

It started with an accidental but long conversation with the "presumably sick and tired administrator" mentioned at the outset. The subject had swiftly turned to the lack of continuity between Basic Algebra and Intermediate Algebra and I came up with . . .

[To be continued]

This entry was posted on Friday, May 29th, 2009 at 5:25 pm and is filed under Uncategorized. You can follow any responses to this entry through the RSS 2.0 feed. You can skip to the end and leave a response. Pinging is currently not allowed.

## Leave a Reply