Notes From The Mathematical Underground

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A Tragedy In Seven Acts

Act I We need to impart some knowledge of some particular piece of mathematics so we prepare a lesson. Perhaps we even give the *content* some thought. Eventually, whether warranted or not, we feel comfortable.

Act II We present the lesson but, for the students for whom this is new stuff, this is not at all as comfortable a situation as it is for us and the lesson is a failure.

Act III So, we yell: "*OK. Never mind all I said before.* ***JUST DO AS I SHOW YOU***" and we proceed to *show and tell* how to answer particular questions.

Act IV Since for the students this is not backed up by *any* understanding of the subject matter (as it is, presumably, for us), it is total voodoo and they have no alternative but to *memorize*: a) how to do whatever we tell them to do *and* b) the questions this is to be done onto. But, no matter how much ensuing *drill*, they give a lot more attention to a) than to b).

Act V Sooner or later, on the exam, on the final exam, on an eventual placement test, … the students flunk.

Act VI This is because they failed to "recall" either a) *how* they were supposed to carry out the voodoo or, more probably, b) *what* voodoo this particular question was calling for.

Act VII Because of that, we concentrate on some refinement and/or alternative to Act IV with no more attention given to the piece of *mathematics* that was involved in Act I.

For a long time now, I have been advocating in these pages what I would like to call **Content Analysis**, that is an examination of the *content* that we teach though, I must say, there is still precious little of it to be found anywhere. What we do get instead is a lot of is educando. Consider the, far from atypical, example

of the Springfield Township School District, a reasonably well-to-do suburban school district in Montgomery County, PA (Lilienthal, 2001).

> The Springfield Township School District has spent the last year designing math standards through seventh grade […]. The math standards focus on extending students' approaches to operational processes to develop critical thinking, as well as computational skills. […] A team of teachers and administrators set out at the beginning of the year to design three to four assessments per grade level that would develop these skills.

It did not augur well, but since I thought I might get some of these "standardized" students in my classes in five or six years, I thought I ought to continue reading. A fourth-grade teacher said,

> With these standards, the district wants to foster an environment where students can begin to think "extensively and logically" about mathematics. […] When children are encouraged to do this, they benefit in a number of ways. They are more motivated, more able to adapt different approaches to different problems and more involved in the decision-making process.

A second-grade teacher said that she "endeavors in her class to extend her student's thinking beyond recall questions to get at how they work out a problem." She then got a bit more specific and said

> Prior to asking these primary recall questions, I asked them questions, like: "How do we solve this problem?" […] "How can we solve this problem differently by getting the same answer?" […] "What strategies did you use to get the answer?" Many students can verbalize or show the math problem, but it's very difficult for them to articulate how they went about their math thinking.

Another fourth-grade teacher "emphasized this extension of student thinking should not be in place of teaching mathematical computations." He said that "Computations cannot be replaced and it is important. But we have to learn to go beyond computation and tap into the important critical thinking and problem solving abilities based on reality."

To be candid, I have no idea of what any of ~~this~~ the above means in actuality. And, I do not think it is the reporter's fault. Some of my colleagues speak that way, and I do not understand them any better.

But, should you think that this is intended only for lower grades, have no fear. The Assistant Superintendent has "been working with administrators to hire an external consultant to implement a similar initiative in the high school." And then, we get their students … and *we* do the exact same kind of things to them under the various names of remediation, developmental education, etc. And, when they themselves become teachers, the circle closes.

On the other hand, I recently came across an article (Haehl, 2001) that impressed me very much as it started with the proposition that "teaching skills through applications and conceptual activities is how to make skills more relevant, therefore understandable, while putting depth into our 'mile-wide, inch deep' curriculum." The author went on to point out that "[w]e hang on to a proven failure, knowing year after year and decade after decade that students who were successful in prerequisite courses do not remember the material." It is rare indeed to see this kind of candid evaluation of what we are doing. The author then proceeded to give examples of the kind of "activities" she used to "focus on concepts" rather than deal in generalized abstract educando which, again, is rather rare.

In her first example, she says that many students don't know that " means 7 four times" and suggests to [h]ave [them] lay out a grid of objects and determine an efficient way to count the objects (like rearranging the rows by moving objects from bottom rows to upper rows until there are 2 rows with 10 objects per row, and the third row has 8 objects). Then, have a student work with fake money and "pay" $4 to each of 7 people and determine how much was paid out. Abstract from there.

Yet, while, mathematically, the above is unassailable, this best-intentioned author does not seem to realize how enormous the amount of mathematics she packs into this *one* "activity" is. Not to mention what she leaves out. Of course, this may be only *one* activity on the matter of multiplication or she may be thinking that going into any more detail would just confuse beginners.

~~However~~In any case, given a particular student body, this example raises at least two very general issues: the *size* of the allowable **conceptual gaps** and the *order* in which the concepts are to be encountered, to which I will refer as **conceptual sequencing**. For example, one question is how many concepts are involved in this "one" activity and another is what other activities had to precede this one in order for it to *make sense* and, therefore, to be successful.

So, while it is not my intention to take Professor Haehl to task for not dealing with these issues--after all, this was not the point of *her* article, I would

like to take this activity as a starting point to develop a bit of Content Analysis from the point of view of conceptual sequencing. In doing so, I shall find myself completely in line with what she recommends and, roughly, I will only suggest that her notion of "activity" really ought to be that of a "sequenced set of activities." Indeed, the above activity is really a *set* of at least four activities.

Activity A is to lay out a 4 by 7 array of objects.
Activity B is to count the objects in an array.
Activity C is to "pay" $4 to each of 7 people.
Activity D is to count how much was paid out altogether.

And, aside from the matter of why the result of D should be the same as the result of B, it certainly must be assumed that the students already know a) what a *grid* is and b) what counting *means*.

While the first can be rapidly disposed of (even though the notion is certainly not trivial), certainly the second ~~must~~ needs to have been ~~already~~ addressed~~.~~ beforehand: We may mean counting *ordinally*, i.e. reciting "one, two, three, …" as we point at the objects, and we will indeed recite "twenty-eight" when we point at the last object. Or, we may be counting *cardinally*, that is putting the array in 1-1 correspondence with some standard set organized in some particular way and whose very organization tells us its name as in 2 TEN & 8 ONES. Note that:

Activity A requires the ability to count up to seven.
Activity B requires the ability to count up to twenty-eight.
Activity C requires the ability to count up to four.
Activity D requires the ability to count up to twenty-eight.

And, even though the notion of counting was not made explicit, it *does* have a bearing on the understanding of " " as, for instance, even very young children can perform Activities A or C since they need only be able to label "small" sets, i.e. sets in 1-1 correspondence with numeric symbols while for Activity B or D they need to know at least some continuation of the "song" for "large" sets, i.e. sets whose names are composites of these symbols. My point, though, is that the activity that consists of "rearranging the rows by moving objects from bottom rows to upper rows until there are 2 rows with 10 objects per row, and the third row has 8 objects" is a *base TEN* counting activity and, as such, is not at the heart of the concept of multiplication. In other words, the activity ought to deal with the concept of multiplication *using* a concept of counting already dealt with.

It is only when we pay close attention to conceptual sequencing that we have a chance to see things from the *students' viewpoint* since the latter lack the

confidence to *jump* from one thing to the other. And, it is clearly necessary to even think of "reinforcement."

Conceptual sequencing also has to do with how we *introduce* concepts since they usually have more than one side. For instance, multiplication raises the issue whether to present it *initially* as **additive power** as in " means 7 four times" or as **cardinal of a Cartesian product** as in a " grid of objects."

Comments, criticisms and rebuttals are very welcome and should be sent to:

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