Part I

Numbers Specified *Directly*, and *Indirectly* By Equations
Arithmetic seems to have originated some four or five thousands years ago when, one may imagine, Sumerian merchants were faced with the problem of accounting for more goods and money than they could handle personally. The solution was to represent the goods in the warehouse and the money in the safe by various scratches on clay tablets so that they could see the situation their business was in without the inconvenience of having to go to the warehouse and/or to open the safe.

As time went by and businesses grew more and more intricate, the bookkeeping system had to become powerful enough to accommodate the new needs. Eventually, about six hundred years ago, Double-entry Accounting was invented, part of which amounted to dealing with signed-numbers. At about the same time, Algebra also developed signed-numbers but as an attempt at systematically undoing what we do in Arithmetic.

We shall thus recreate, with a bit of poetic license but from a mathematical point of view, the way “accounting” might have evolved over the centuries and, more or less in that order, we shall account for:

- Money on the counter,
- Money changing hands over the counter,
- Goods in the warehouse,
- Goods changing hands over a counter,
- Goods exchanged for money (namely buying and selling).