coefficient
original
exponent

Chapter 1

Multiplicative Powers

1.1 Repeated multiplication/division

Given a number a, we shall often have to multiply or divide it by a number of copies of some other number x

1. We begin by discussing the corresponding language.

- $a\ (x)^{+3}$ is to be read as a *multiplied* by 3 copies of x
- $a\ (x)^{-3}$ is to be read as a *divided* by 3 copies of x

  where
  - the number a is called the **coefficient**,
  - the number x is called the **original**
  - the number, $+3$ or -3, is called the **exponent** where
    - the counting-number 3 indicates the number of copies to be made of the original
    - the sign, $+$ or $-$, indicates whether the coefficient is to be *multiplied* or *divided* by the copies

Occasionally, the exponent will turn out to be 0, but, even in that case, we will continue to have

- $a\ (x)^{0}$ is to be read as a *multiplied/divided* by 0 copies of x

In this last case, we thus have

$$a\ (x)^0 = a$$

But then, by comparison with

$$a \cdot (+1) = a$$

we can conclude that

$$(x)^0 = +1$$

positive third power
negative third power
zeroth power

2. When replacing a and x by signed numbers, it is safer to enclose them within parentheses. For instance, with $a = +7$ and $x = +5$, we write:

$$\begin{aligned}(+7)(+5)^{+3} &= +7 \textit{ multiplied} \text{ by 3 copies of } +5\\ &= (+7)\cdot(+5)\cdot(+5)\cdot(+5)\\ &= (+7)\cdot(+125)\\ &= +825\end{aligned}$$

and

$$\begin{aligned}(+7)(+5)^{-3} &= +7 \textit{ divided} \text{ by 3 copies of } +5\\ &= \frac{+7}{(+5)\cdot(+5)\cdot(+5)}\\ &= \frac{+7}{+125}\\ &= +0.056\end{aligned}$$

and

$$\begin{aligned}(+7)(+5)^{0} &= +7 \textit{ multiplied} \text{ by 0 copies of } +5\\ &= +7\end{aligned}$$

Altogether, we have

The code:	to be read as:	says to write:	gives:
$(+7)\ (+5)^{+3}$	+7 multiplied by 3 copies of +5	$(+7)\times(+5)(+5)(+5)$	$(+7)\cdot(+125)$ $= +875$
$(+7)\ (+5)^{-3}$	+7 divided by 3 copies of +5	$(+7)\div(+5)(+5)(+5)$	$\frac{+7}{+125}$ $= +0.056$
$(+7)\ (+5)^{0}$	+7 multiplied by 0 copy of +5	+7	$= +7$

1.2 Powers

Of course, + and +1 and, even more so, +1 tend to go without saying.

1. In particular, the *coefficient* +1 usually goes without saying and then

- $(x)^{+3}$ is called the **positive third power** of x
- $(x)^{-3}$ is called the **negative third power** of x
- $(x)^{0}$ is called the **zeroth power** of x

gauge power

In other words, powers can be looked upon as repeated multiplications involving the coefficient $+1$ with the latter going without saying.
However, it will be more convenient for us to use both the coefficient $+1$ and the coefficient -1. Thus,

- $+(+5)^{+3}$ will mean $(+1) \cdot (+5)^{+3}$
- $-(+5)^{+3}$ will mean $(-1) \cdot (+5)^{+3}$
- $+(+5)^{-3}$ will mean $(+1) \cdot (+5)^{-3}$
- $-(+5)^{-3}$ will mean $(-1) \cdot (+5)^{-3}$

and we will call these **gauge powers**.

2. When it is the *exponent* which is equal to $+1$, it is less of an issue to let it go without saying since

- $(+7)(+5)^{+1}$ is to be read as $+7$ multiplied by 1 copy of $+5$, that is $(+7)(+5)$,

while

- $(+7)(+5)$ is to be read as $+7$ multiplied by $+5$, that is $(+7)(+5)$

which is exactly the same as above.
On the other hand, the exponent -1 can never go without saying.

3. When replacing x by a negative number, for example -5, in a power, one should be careful that

-5^{+4} does *not* work out to the same number as $(-5)^{+4}$

Indeed,

- in -5^{+4}, the $-$ stands for the coefficient -1 and the 5 stands for $+5$ and is the original of which the copies are to be made. In other words, we have:

$$\begin{aligned}-5^{+4} &= (-1)\ (+5)^{+4} \\ &= (-1) \text{ multiplied by 4 copies of } (+5) \\ &= (-1) \cdot (+5) \cdot (+5) \cdot (+5) \cdot (+5) \\ &= -625\end{aligned}$$

- in $(-5)^{+4}$ the coefficient is $+1$, going entirely without saying, and -5 is the orginal of which the copies are to be made. In other words, we have:

$$\begin{aligned}(-5)^{+4} &= (+1)\ (-5)^{+4} \\ &= (+1) \text{ multiplied by 4 copies of } (-5) \\ &= (+1) \cdot (-5) \cdot (-5) \cdot (-5) \cdot (-5) \\ &= +625\end{aligned}$$

On the other hand,

-5^{+3} *does* work out to the same number as $(-5)^{+3}$

parity

Indeed,

- in -5^{+4} the $-$ stands for the coefficient -1 and 5 standing for $+5$ is the original of which the copies are to be made. In other words, we have:

$$\begin{aligned}-5^{+3} &= (-1)\ (+5)^{+3} \\ &= (-1) \text{ multiplied by 3 copies of } (+5) \\ &= (-1)\cdot(+5)\cdot(+5)\cdot(+5) \\ &= -125\end{aligned}$$

- in $(-5)^{+3}$ the coefficient is $+1$, going entirely without saying, and -5 is the original of which the copies are to be made. In other words, we have:

$$\begin{aligned}(-5)^{+3} &= (+1)\ (-5)^{+3} \\ &= (+1) \text{ multiplied by 3 copies of } (-5) \\ &= (+1)\cdot(-5)\cdot(-5)\cdot(-5) \\ &= -125\end{aligned}$$

This has nothing to do with the sign of the exponent and the same would hold true with a negative exponent.
What is involved here is the **parity** of the exponent, that is whether the exponent indicates an *odd* number or an *even* number of copies. This will turn out to be extremely important when we investigate power functions.

1.3 Roots