

# Calculus anyone?

A sick column by Alain Schremmer.

## ALL QUIET ON THE CALCULUS FRONT

In the March 1991 issue of **UME Trends**, there appeared a report by Paul Zorn on a "recent NSF-supported conference in San Antonio, organized by Andrew Gleason and Deborah Hughes-Hallett from Harvard, Jerry Uhl and Horacio Porta from Illinois, and Shair Ahmad from the University of Texas in San Antonio [in which] the mathematical content of calculus courses, rather than pedagogy or technology, would be a primary focus."

The "high connectivity of calculus's content, pedagogy, and applications" was affirmed as some sort of given Holy Trinity, but the conference seems to have been limited to a non-theological discussion of Our Calculus Who Is In Heaven, namely to a discussion of dogma, catechism and dependence of Church and State.

What interested me in Zorn's article was indeed that, as usual, all references were to THE calculus. We talk about algebra, topology, analysis, arithmetic, etc, with or without capital, but we do not prefix any of them with the definite article. Similarly, try "topology reform", "homology reform", "trigonometry reform", etc

And indeed, that there might be something flawed about the mathematical underpinning of THE calculus does not seem to have come near anyone's mind.

Instead, the usual thrashing about which the "real problem" was continued unabated. Is it the contents? No. You will be relieved to learn that "no blue ribbon syllabus for the calculus of the future will emerge from the San Antonio conference". Is it the pedagogy? No: "For Ed Dubinsky, from Purdue University [and Editor of **UME Trends**] the content (in the sense of formal syllabus) of a course is almost irrelevant: what counts is the (often very different) 'content' that students receive".

Now, as readers of this column, if any, know, there is more than one way to cook the calculus. You can cook it à la Bolzano-Cauchy-Weierstrass, à la Robinson, whether or not in the "decimal" version (the W. Freed brand is highly recommended), à la Lagrange (our own brand is not to be disdained). So, could someone out there convene a conference with still a "new twist"? That is a conference that would be a bit interfaith and where the various ways to calculate, or to invoke The Calculus if you prefer, would be compared and discussed.

It had been with great regret that I had missed Sheldon P. Gordon and Deborah Hughes Hallett's talk in Dallas, so it was with great trepidation that I

started on their article, *Calculus Reform and Its Implications for Two-Year Colleges* in the Spring 1991 issue of the **AMATYC Review**. We are talking big guns here. Hughes Hallett's grant is the famous monster grant awarded to Harvard to save calculus, the US and the Queen. It didn't start too well:

PAGE 50. "Calculus had often been described [...] as 'one of the greatest ...' Obligato sostenuto. "On the other hand, ..." Cymbals and funeral drum rolls. "At the same time, calculus textbooks ..." Pizzicati. "Over the last few years, leaders ...". Thunderous, Mahlerian crescendo. "The rapid rate of technological growth ..." Andante. "To address these problems, the National Science Foundation [gave us close to 2 megabucks.]" Radetski march.

PAGE 51. Things now become wildly controversial, if not starkly revolutionary: "Five major thrusts: It de-emphasizes the current stress on manipulative skills ..., It presents a more intuitive approach ..., It introduces more modern mathematical ideas, It includes a wider variety of applications ..., it incorporates the use of appropriate technology ..." This, Ladies and Gentlemen, if perhaps not sufficient to write 2 megabuck best sellers (but this remains to be seen), obviously is how to write 2 megabuck NSF grants. Whether this is going to solve the problems of my students, I doubt. But, no doubt, Pr. Sheldon will tell us in Seattle how it helped his students.

PAGE 52. "The rule of three [...]: whenever possible, all concepts and methods in calculus should be presented in three ways—geometric, numerical and symbolical". It is also understood that we are all in favor of apple pie and

We are sorry to report that, at this point, Professor Schremmer was found lying on the floor, clutching convulsively a copy of Lagrange's *Leçons sur le calcul des fonctions*. We were able to revive him with a differential injection and to rush him to the nearest advanced calculus course. He will resume his column as soon as possible. [Office mate]

Where was I? No, I can't take it any more. Suffice it to say that, on page 54 they "have attempted to very thoroughly delineate the difference between the notion of the **global** and the **local** interpretations of the behavior of a function. Thus, we have carefully separated the ideas of the derivative at a point (whose value tells you about the function's local behavior) and the derivative as a function (whose value across an interval gives you global information). Further, we have given major emphasis to the idea that the derivative at a point gives the best linear approximation to a

function at that point. In fact, we have utilized this theme heavily throughout the development of the differential calculus to make it clear to the student that it is an important tool of mathematics. " Well, as my great aunt used to say, better to read this than being blind.

By the way, the report of the San Antonio conference mentioned that "[t]he theme of approximation, it was agreed, is central to the calculus—what it is and what it does", yet, significantly, this role was mentioned in only this one paragraph of an otherwise fairly discursive article.

In the March 1991 issue of the **Monthly**, safely ensconced in the Teaching section, I find an article on calculus by Murray H. Protter on *The Self-Paced Calculus Program at Berkeley*.

This one is quite predictable as witnessed by the section headings: How the self-paced course works, Grading and units credit, Record keeping, Who should take self-paced calculus? Cost, Other courses. If that's the kind of things you are interested in, then by all means ... It should be noted that Protter is the author, with the late Morrey, of a calculus book that was well known twenty years ago. For all I know it might still be to this day.

But if I mention this article, it is because of quite an arresting passage in the introduction:

"[I]n 1971, physicist Robert Karplus and I introduced a self-paced course that combined first-year physics and first-year calculus. We wanted to remedy the justifiable complaint of many physics teachers that their students never seem to learn the mathematics needed for understanding principles of physics until after these principles are taught in the physics course. Physics instructors frequently must teach bits of mathematics to students who otherwise would be bewildered by the physics being presented.

*By interweaving mathematics and physics we achieved in our disciplinary self-paced course a program that taught both subjects in a logical and consistent manner. The course was taught for two years and then dropped because combining physics and mathematics in one course put too heavy a burden on the students."*

I must assume that the combined course included the hours of both the original calculus and physics courses. So, given all the hoopla about applications, I would very much like to learn about the nature of the overburden. I think that there is something of very great interest here. Has anyone had any similar experience?

The second thing which I find puzzling is the situation that Protter describes in the first paragraph quoted here: Their teachers' complaint notwithstanding, physics students are said to "*understand[...] principles of physics*" with no more than the "*bits of mathematics*" without which

they "*would be bewildered by the physics being presented*". So what is calculus needed for?

The third point which I find interesting is that the "*logical and consistent manner*" in which the course was taught seems only to have made the burden insufferable. Is this to say that if a little bit of knowledge is bad, more knowledge is even worse or is this to say that logical consistency does not necessarily make good solid common sense?

## CALCULUS SURVEY

I would like to make a survey of what the readership attitude is regarding the calculus crisis (if any) and the various calculus initiatives (if any).

First, I would like the readers of this column (if any) to let me know what questions should be asked. Upon receipt of the questions (if any), I will publish them and wait for the answers (if any). Then, I will publish and argue with the answers. Then I will publish the counter arguments. Then ...

## QUESTIONS

1. Why should the definition of continuity, a property of  $f$ , be followed with the definition of the derivative, a function, rather than with the definition of differentiability, a stronger property of  $f$ ?

2. Or, to put it another way, why should Differential Calculus be based on the notion of derivative in dimension 1 when it must be based on that of differentiability in all other dimensions?