Calculus anyone?

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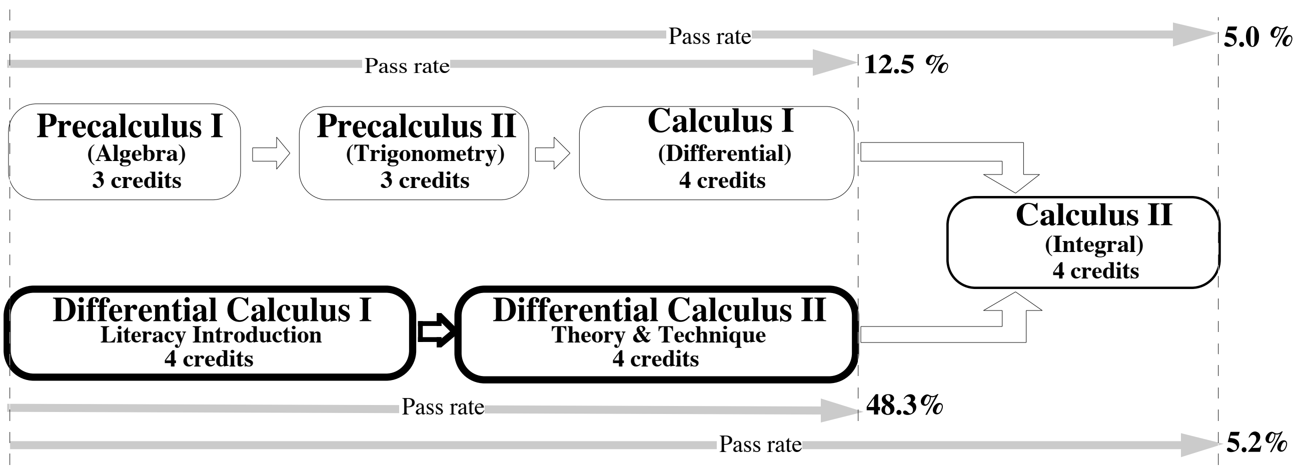
The news of the impending demise of this column have been greatly exaggerated. Aside from my personal arrogance and obstinacy, each of which, alone, would ensure infinite longevity of this column, an interesting fact has emerged.

As has perhaps not gone completely unsuspected, I have been involved for some years in a calculus renewal effort based on the use of … Lagrange's approach to differential calculus, that is, based on the systematic use of local polynomial approximations because it has, among various advantages, the merit of allowing a seamless integration of the entire precalculus with the differential calculus.

The new two-semester sequence, which I shall refer to as the Lagrange sequence, started being offered in Fall 1988 and has been offered ever since. But we were concentrating on developing the materials and we had not paid too much attention to matters of evaluation.

Then, last year, some of our distinguished colleagues, both inside and outside the mathematics department, launched an attack on the Lagrange sequence by way of letters to the editor in the students' newspaper. The main thrust of the attack was the contention that, while the Lagrange sequence might indeed produce more students, at Calculus I level, these students were not necessarily prepared for Calculus II. We argued that, in any case, not allowing students to reach Calculus I level on the grounds that they could not reach Calculus II level was hardly defensible but, somehow, the argument failed to make much of an impact as the impression remained that the Lagrange approach did not produce a real Calculus I level.

Eventually, the college's Office for Institutional Research investigated the 654 students who had entered both sequences between Fall 1988 and Fall 1989. The percentages are rather stunning:



Thus, the chances of a student to achieve Calculus I level are almost four times in the Lagrange sequence what they are in the traditional sequence and do not affect her/his chances to achieve Calculus II level. And, that the levels reached in the Lagrange sequence and in the traditional sequence were comparable was established by the passing rates in the conventional Calculus II which were respectively 64.3% for students coming from the Lagrange sequence and 80.6% for students coming from the traditional sequence.

One likely reason for this difference might be that we had been concentrating on developing and refining the first semester materials while the text for the second semester had remained practically unchanged since the beginning and thus the difference might be shrunk. Another factor might be that the students in the traditional sequence are using the same book in Calculus I and II. Thus, the approach used in Calculus II does not take advantage of much of the knowledge acquired in the Lagrange sequence. A third factor might be that the attrition rate *between* Calculus I and Calculus II seems to be higher for the students coming out of the Lagrange sequence and does not seem highly correlated with the grades obtained: It just seems that students a priori determined to go through calculus tend to enter the traditional sequence while the Lagrange sequence might attract students more uncertain with their future plans.

So, what now?

Clearly there is a need to develop a text for Calculus II that takes advantage of what has been developed before. Since integration techniques already tend to be deemphasized, it is perhaps the time to shift the emphasis to the global.

To take only one example, the concept of local approximation having been already acquired, and the problem of passing from the local to the global having already been introduced, it becomes fairly natural to deal with the problem of global approximations. Another natural thing is to ask whether the successive polynomial approximations of $f(x_0+h)$ get closer to $f(x_0+h)$ and even if they might converge. In other words, this introduces *series*.

The Fundamental Theorem of Calculus can be introduced, very naturally as the solution of an initial value problem extended to an interval:

And the integral itself can be introduced via its Riemann sum approximations very much in the style that I introduced the exponential function in my previous column.

But, really, for the largest majority of the students who currently need to take one year of calculus to meet some academic requirement, even such a modified Calculus II is not appropriate and a one semester course in Dynamical Systems would be an alternative to integral calculus better suited to the needs of those students oriented toward sciences other than Mathematics, Physics and Engineering.

There are two aspects to the integral calculus which, in the one-dimensional case, are merged and thus usually insufficiently distinguished: The *measure theoretic* aspect with the definite integral defined as limit of finite Riemann sums and the *antiderivative* aspect with the antiderivative defined as the solution of $y' = f(x)$ with f given with some condition which leads to the notions of potential and exactness of forms. To quote Picard, "*Integral Calculus was born the day one asked the question: given f(x), does there exist a function whose derivative is f(x), in other words a function which satisfies*

$$\frac{dy}{dx} = f(x) \text{ "}$$

Indeed, while essential in mathematics, physics and engineering, where one needs to compute integrals over areas and volumes, the measure-theoretic aspect of the integral calculus in other sciences is of limited applicability and the type of applications most likely to be encountered by "just plain folks" is more often than not in the form of Initial Value Problems.

Such a course might or might not be construed as a course in modeling. What is important here is that it would be a completely natural continuation of Lagrange's approach.

FOCUS ON CALCULUS

In an earlier column, à propos an article by Sheldon P. Gordon and Deborah Hughes Halett in the Spring 1991 issue of the ***AMATYC* Review**, I had written that while their ideas were "*perhaps not sufficient to write 2 megabuck best sellers (but this remains to be seen), obviously [this]* is *how to write 2 megabuck NSF grants*."

I am happy to report that it begins to look like this *is* going to be a 2 megabuck best seller. We are about to witness a campaign that will make the German campaigns through Belgium look like molasses. John Wiley and his sons are not sleeping: A Conference on the Teaching of Calculus to be held at Harvard University, a Newsletter called **FOCUS ON CALCULUS** just to begin. A couple of dozens of "test sites". This is seed money well employed. Enough to make Iacocca jealous. Enough to make anyone jealous, including myself.

I was going to indulge in a few quotes from the Newsletter, even at the risk of another heart attack but, of course, you all received it. So, what more is there to say?

CALCUDUKE

Another project (over $900,000 from NSF) which is generating a lot of hype is the project at Duke University. Another Big Science project. This time, my source is an article in the February 28, 1992 issue of Science entitled **The Calculus of Education Reform**.

The Duke project focuses—Oops, this word is now copyrighted by the Harvard-Wiley consortium—on high tech pedagogy. "*In one lab, for example, students tackled world population, exploring which of two mathematical models best fit population growth to date. They plotted real data against time—or rather, they ordered their computers to plot the data swiftly for them. Then, they graphed the same data on semilog axes, to see is the points took the shape of a simple exponential in which the population doubles as a steady rate over time—and which graphs as a straight line on semilog axes. The colored points lighting up the computer screen clearly showed that a straight line model wouldn't do—the population was increasing too rapidly. So the students turned to the second model, an even more steeply rising super exponential function that seemed to fit the data better. They then used the data to estimate the parameters* *of this function and also algebraically solved its differential equation.*

The whole idea is to reinforce two central concepts, [...]. First that the slope of a curve equals rise over run. And second, that a derivative is a rate of change, which can be expressed in a graph, in algebraic symbols, or as numbers."

Stunning! But notice the many connections to what I was writing about earlier on and in previous columns. So, this is the mother lode from which many columns are to come.

In the meantime, readers are invited to meditate on the profound meaning of the above excerpt.

Another, very interesting side to the issue, also dealt with rather extensively in the Science article, is the students' reaction to the Duke Project and Duke's reaction to the students' reaction.