Let's Face It!

An argumentative column by A. Schremmer

Given indeed my argumentative nature, I had promised myself never to comment on articles appearing in this newsletter. I wanted to avoid the "exchanges" which you occasionally find in the press of opinion. (As if any press ever was objective. But this, as Kipling used to say, is another story.) Anyway, human nature, particularly mine, being what it is, here I go.

I just want to propose a criterion to decide when to use calculators or, more generally, computer algebra systems and/or graphing packages. I will use the term system to cover all. Actually, of course, I mean a criterion to decide when to allow students to use systems. But let me get away from systems for a moment.

Whenever I am working on a problem, and I still do once in a while, I fiddle, I tinker, I draw pictures, even if only an oval to represent a set or a parallelogram to represent a space. In other words, I am doodling while I am thinking about it. Eventually, I get an idea, usually not a very good one, but one worth putting to some test and that is when I start calculating. Psychologically, at least for me, calculating means getting in an automatic mode, that is to say a non-thinking mode. Once the calculation is finished, I start thinking again to look at what came out of the computation. Thus, I would not mind having the calculation done by someone else and, in fact, I have a couple of friends I love to work with because they love to calculate. As the French say, tous les goûts sont dans la nature. Nevertheless, I would prefer to have some calculating system to do the work.

(If it seems that I exhibited a certain amount of contempt for those of us who calculate, nothing could be farther from the truth. I don't like to calculate because I am not good at it but I am perfectly aware that calculating can be tinkering too. To give you an example, my wife (of course, as we all know, women ...) who is the real mathematician in the family (watch it, he is going to tell us that she is the one who balances the checkbook) likes to tinker in Sobolev spaces. She writes inequalities after inequalities just like I draw ovals after ovals. I can't believe it. She assures me that, for her, deriving these inequalities is tinkering. Maybe then, what she should do is to use a graphing system to do her grundge work1.)

In any case, here is my criterion.

SCHREMMER's CRITERION: Systems should be employed only to relieve boredom and/or to provide leverage.

Now, about thirty years ago, Z. P. Dienes observed that playing children followed a cycle which he proposed as a basis for learning [1], [2]. First children play with concrete objects according to given rules which they follow rigidly while they observe the effect of the rules on the objects. Then, they start relaxing the rules, more or less randomly, and with the emphasis remaining on the objects. But, after a while, their attention progressively turns to the rules themselves which then become the objects of the game. And the cycle starts anew: now the children are playing with the rules according to superrules which, after a while, they start relaxing. It is worth noting how the children

oscillate between a need for the reassurance of hard and fast rules and a need for breaking these rules.
We might say that they are using sets of operators. It is indeed not difficult to get them to see things systematically that way. In fact, my previous column owed a lot to Dienes.
About twenty five years ago, as a Fullbright Fellow at the University of Pennsylvania, I decided to develop some of Dienes' schemes. I asked for third grade class that I would meet everyday for an hour. The School of Education was associated with an elementary school and the principal there gave me a third grade, lowest track, the reason being, I suspect, that there was nothing that I could do to damage them. They didn't know any arithmetic. They were beyond damage, they were hopeless. Or so I was told later.
The culture shock for me was severe but, thanks perhaps to my naïveté, I was able to get the children to play seriously. I don't remember what I had set out to do but I remember that I didn't do it so fascinated was I by the intensity with which the children were playing. It was as if this was the first time that they had ever been playing seriously.
By the middle of the Spring, one of the children, Russell, was not only able to do arithmetic in any base without the aid of the multibase arithmetic blocks (several other could too) but he had set out to write the rules for doing so.
I thought that I could recognize talent when I saw it and I went to see the principal. The principal assured me that I had to be mistaken. This was the lowest track (all black but the principal did not allude to this). The children had all been tested. If Russell had been as I thought, this would have shown on his IQ and he wouldn't have been in this class. I insisted and I suggested that the principal see by himself. The principal recused himself but, presumably because of my credentials, had Russell's file brought out. The principal was right. Russell's IQ was in the eighties. But here was Russell writing base free algorithms for arithmetic!
I am afraid that the tale does not have a happy end. I was a foreigner. I didn't know the ropes. A month later, I had to leave the school. I never saw Russell again. If he is still alive, which is not necessarily the case given where he came from, he must be pushing forty. Sometimes, I wonder what I did to him.
Playing admittedly takes time. So, what do they do to save time? "Mary, don't bother me with your endless questions. Just do as I tell you. It's much faster and much simpler that way. When you get to college, you can ask your instructor."
You would think, or at least, not having been raised in this country, I would think, that there is plenty of time in the first three or four grades. Anyway, the net result of this time saving is what we see in our classes. By that time of course, Mary has learned her lesson: if you know the right rule, any math problem can be done in less than two minutes.
So here is a

COROLLARY. Systems should never be used where they will impede direct experience.

In other words, using systems should never shortcircuit the phase during which the students experience whatever it is that they need to experience.

Observe by the way that we do not memorize the addition tables but that we operate from a certain gestalt. We know that 13 and 8 are 21. What does this say?

You might argue that, as desirable as it might be to foster such a cycle in our classes, administrative constraints make it utterly impossible.
It is always easy to blame the neighbors. Do you really think that what we are doing in, say, our "remedial" classes has much value at all? (We all do exactly the same thing). How many of the entering students who need to take remedial arithmetic end up in calculus? Aren't we hiding behind the administration, the textbooks, the poor working habits that our studnets acquired in secondary school? Bu t what are we doing that is different from what they are doing. Besides, we trained them! How many of your students are going in Early Childhood Education and how much love of mathematics do they carry there?
Finally, will systems, (calcululators, computer algebra systems, graphic packages, etc) help convey the love of tinkering, of exploring, of making sense of what is observed? My answer is a resounding

NO!

Let me put my money where my mouth is. I am taking bets. Any odds you want. Ten years from now (I can't take longer bets because I will be retiring then), the HP 28S will look ridiculously primitive compared to what will be available then. That's not the bet. But the students' mathematiization will be worse than it is today. That's the bet.

References

1. Z. P. Dienes. "Building up mathematics." 1960 Hutchinson. London.
2. Z. P. Dienes. "An experimental study of mathematics-learning." 1963 Hutchinson. London.

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The truth of the matter is, God did not create numbers. Accountants and Kronecker did. She created functions.

1. In the beginning, there was the zero function.
2. The Lord God looked down upon the zero function and She was displeased.
3. And so She said, "Let there be non-zero outputs" and there were constant functions. And they were closed for addition.
4. And She smiled at Her work and it was good.
5. All this was done between the increasing and the decreasing of the first function1.
6. The Lord God did look out over the constant functions and She did become bored2. For She could not stand things that remained unchanging.
7. And so she did say, "Let the output be proportional to the input". And thus were linear functions born.
8. And linear functions had a rate of change.
9. And the rate was good.
10. When She realized that linear functions were monotonic, She did say, "Let half the linear functions be used for counting the bulls, and let them be called the increasing functions."3

11. "And let half of the linear functions be called decreasing, and let them be used only to count the bears because, yes, verily, these functions are evil and should be used as seldom as possible for it is My Word.
12. And she closed the universe again by adding constant and linear functions. So were affine functions born.
13. And the first derivative was created and it was a constant function. And the second derivative was created and it was the zero function.
14. And God told the parabola of the quadratic functions.
15. And God became impatient and created all polynomial functions at once rather than just cubic functions.
16. And then She was good to us because She knew that we couldn't wait for ever for outputs to get large and She created the reciprocal function who has large outputs right at the beginning. And polynomial functions could still be used to approximate the reciprocal function anywhere else.
17. But She added the reciprocal function as a gauge functions and Laurent had his polynomials too.
18. And to close again the universe, She created rational functions.

Etc. Eventually, on the sixth day, She created analytic functions, but, on the seventh day, She too did test.

Added in Proof: she now does.

1 This is obscure as, only constant functions have been created at this point.
2 This is more likely than her becoming confused.
3 This anachronism is difficult to explain.