Let's face it

A true column by A. Schremmer.

In my first column, I warned that I would hold us guilty of debasing mathematics education. Looking back, a few columns later, I realize that I didn't do that. I mean that, rather than looking at what is *globally* wrong with what we are doing, all I did was to make a few *local* criticisms. Rather than to look for the basis spanning the disaster, I discussed a few points.

The reason, though, is clear: The current disaster in mathematics education has not so much to do with mathematics as with education and, coming from, and having been educated in, a country where education does not exist as an academic discipline, I have a prejudice against discussing education and tend to leave this to educators. I am, of course, wrong: Education is too serious a matter to be left to educators.

I decided to look at the matter from a general, ideological point of view. But then, being only a mathematician, and a very mediocre one, I didn't even know where to start. I was reminded of Grothendick's dictum that the hardest thing in a problem is to phrase it in the right category. It didn't help, especially as, years ago, I had lost faith in the universality of category theory as an epistemological tool.

But then I came across an article by Colin McGinn, *Homage to Education*, in the August 16, 1990 issue of the London Review of Books. I was tempted to merely send it in to appear in place of this column. But, while what McGinn discusses does not directly invoke mathematics, it has a particular echo in mathematics. I thus decided to quote and discuss it in a mathematical perspective. In any event, I cannot recommend too strongly that you read the original article which is a review of a book of, and of a book about, R. G. Collingwood. The part in which I am interested here is where he "*spell*[*s*] *in* [*his*] *own way what* [*he*] *thinks Collingwood is getting at here*."

"*Democratic States are constitutively committed to ensuring and furthering the intellectual health of the citizens who compose them: indeed, they are only possible at all if people reach a certain cognitive level..* [...]. *Democracy and education (in the widest sense) are thus as conceptually inseparable as individual rational action and knowledge of the world.* [...]." But what is education? "*Plainly, it involves the transmission of knowledge from teacher to taught. But what exactly is knowledge?* [...]. [*It*] *is true justified belief that has been arrived at by rational means*." [...]. *Thus the norms governing political action incorporate or embed norms appropriate to rational belief formation.* [...]. *The educational system of schools and universities is one central element in this cognitive health service* [...].

The quasi-mathematical language in which this is stated should have a special resonance for mathematicians.

"*It would be a mistake to suppose that the educational duties of democratic state extended only to political education, leaving other kinds to their own devices.* [...]. *How do we bring about the cognitive health required by democratic government? A basic requirement is to cultivate in the populace a respect for intellectual values, an intolerance of intellectual vices or shortcomings*. [...]. *The forces of cretinisation are, and have always been, the biggest threat to the success of democracy as a way of allocating political power: this is the fundamental conceptual truth, as well as a lamentable fact of history*."

However, "*people do not really like the truth; they feel coerced by reason, bullied by fact. In a certain sense, this is not irrational, since a commitment to believe only what is true implies a willingness to detach your beliefs from your desires*. [...]. *Truth limits your freedom, in a way, because it reduces your belief-options; it is quite capable of forcing your mind to go against its natural inclination. This, I suspect, is the root psychological cause of the relativistic view of truth, for that view gives me license to believe whatever it pleases me to believe*. [...]. *One of the central aims of education, as a preparation for political democracy, should be to enable people to get on better terms with reason – to learn to live with the truth*."

The only question then is who is to decide what the truth is. But truth was defined in 1933 by Alfred Tarski. Well, sort of. See [**1**]. The real issue is to start looking at mathematics from a *political* viewpoint rather than from the usual narrow *utilitarian* one. When deciding to teach some thing or the other, the real question is not whether the students will be able to use it in their day-to-day life or in their job but whether it will help them not to be taken in by the people in power[1]. In fact, this political viewpoint is taken by statisticians when they explain why people need to understand statistical inference. But then we all know that they are not mathematicians and that, in any event, statistics should not be taught by mathematicians. But why do mathematicians care so little about logical inference? I mean in real life. Of course, it is a matter of fact that, in real life, mathematicians, and even logicians, are not discerningly more rational than it would appear they ought to be.

So, Ladies and Gentlemen, scholars (and educators) that we are, what do we prepare our students for? What do we do with our students? In algebra we teach them

[1] Does this statement have its place in a mathematical newsletter? By the way, anarchism does not mean disorder but, rather, the avoidance of any concentration of power.

how to factor trinomials by inspection, in Precalculus how to use Descartes' rule, in Trigono-metry how to solve triangles, in Differential Calculus how to optimize a function by killing its derivative, in Integral Calculus how to compute integrals by quadrature, in Differential Equations how to solve in closed form or by power series, etc. What do all these have in common? That, when we write the corresponding exams, we must be careful to choose problems where these "methods" will work. How is this to bring about the "*cognitive health*" required by democracy? I contend that not only do we make a mockery of mathematical education but that, as a consequence, we are traitors to democracy.

And, here, I will again lapse into episodism. I recently had the occasion to argue educational matters with a Ph.D. in Applied Mathematics. My interlocutor had started by asking me what 2^3 was. Identifying with most of my students, I replied 6. My interlocutor recovered almost instantly and said "OK, what are the four operations?" and he wrote on the board

$$2+3 = 5$$
$$2-3 = -1$$
$$2x3 = 6$$
$$2\div 3 = 2/3$$

and finally

$$2E3 = ?$$

I could see that he was going to say that the answer couldn't be 6 since 6 had already been given by multiplication! Of course, by that argument 2x2 cannot be equal to 4 since 2+2 already gives that value. But I chose a sneakier development. I pointed out that E is not a binary operation, not even on R: $-2E\frac{1}{2}$ = ? The point of this is not that my interlocutor hadn't realized it but that he had never *wondered* about it, (Had he done so, he would have instantly realized it.) He had never wondered about the relation between powers and operations. He had been perfectly happy with his little trick for supposedly convincing students that $2^3 \neq 6$. For the second round, I used my standard opening and asked him to defne multiplication**1**: After he defined 2 x 3 as 3 + 3 and 2 x $\frac{1}{3}$ as $\frac{1}{3} + \frac{1}{3}$, I countered of course with $\frac{1}{3}$ x 2. Without the slightest hesitation, he invoked commutativity. After we got that straightened out, I moved to $\frac{1}{2}$ x $\frac{1}{3}$ which he countered with: from $\frac{1}{2}$ x 2 = 1 and $\frac{1}{3}$ x 3 = 1, $\frac{1}{2}$ x $\frac{1}{3}$ x 6 = 1 and therefore $\frac{1}{2}$ x $\frac{1}{3}$ must be equal to the inverse of 6, $\frac{1}{6}$. He clearly felt that he had checkmated me. But if a student is capable to be really convinced by this argument, then she ought to be in abstract algebra. In a remedial arithmetic course, the argument is a fraud. Here was a person who had obviously given quite a bit of thought to the *teaching* of arithmetic but, because he hadn't thought too much about arithmetic *itself*, had somehow completely missed the political question which was to decide what the *true* definition of multiplication *is* rather than to make do with one which *can be made* to do the job**2**. But the means determine the end.

What troubles me is not so much that little real innovation is forthcoming but that so little questioning is going on, say at the annual meeting of the AMATYC. We submit our papers, our workshops, our mini-courses, that is, in short, our *answers*. Where do we ask the *questions*? The fact is that innovation can only proceed from questioning. What I find among mathematics teachers**3** is primarily a lack of desire to question the "truths" that they have received, or think that they have received.

It is certainly not given to all of us to be radical. It is even more certain that most institutions do not encourage innovation**4**. Yet, I find that we are, by and large, complacently conformist (as in "since we have got to conform, we might as well be proud of it"). But we don't necessarily have to take on the students, the dear colleagues (and the department head), the administration or, in some states, the governor. Aren't there ways in which we can create a subversive atmosphere in the classroom, that is one that is just a bit more conducive to rational reflection?

For instance, why are we teaching from the usual textbooks? Do we really have to? Will our administration fire us if we don't? And if we need a textbook, why can't the textbook merely present the body of knowledge in question? Why does it have to be intimately mixed with a pedagogical treatment? Why can our colleagues in the English Department discuss, and sometimes even ask their students to discuss, "primary" texts while we cannot? I am not advocating using Euclid, L'Hôpital, etc.—although we could be, we are doing, much worse—but why can't *we* write primary texts to be *discussed* as such?

1. J. Barwise and J. Etchemendy. "The Liar." 1987 Oxford University Press. New York.

1 See my column in the Winter 1990 issue.

2 Beyond that, to be honest, I have no idea of how to teach arithmetic to adults in a manner designed to help them "*get on better terms with reason*" in forty-two hours. In fact, I believe that arithmetic is not where to start an adult education. More about this in a future column.

3 At *any* level

4 Innovation should not be encouraged because it is good in itself but because, given the status quo, we have nothing to lose.