

## Chapter 8

# Add-On Operators

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**Add-on operators** are “super-functions” in the sense that they are to functions what functions are to numbers: given an **input function**, an add-on operator returns an **output function** in terms of an **add-on function**:

$$\left[ x \xrightarrow{INPUT} INPUT(x) \right] \xrightarrow{ADD-ON\ ADD-ON\ FUNCTION} \left[ x \xrightarrow{OUTPUT} OUTPUT(x) \right]$$

where

$$OUTPUT(x) = INPUT(x) + ADD-ON\ FUNCTION(x)$$

### 8.1 Constant Add-On Functions

The simplest add-on operators are when the add-on function is a *constant function* because the output function just returns the output of the input function with the coefficient of the constant add-on function added on.

1. When the functions are specified by *tables*, things are pretty simple.

**EXAMPLE 1.** When we shop online, there is often a fixed add-on “Shipping Charge”, say \$4.25, which we represent by the *constant* add-on function:

$$x \xrightarrow{SHIPPING_{4.25}} SHIPPING_{4.25}(x) = 4.25$$

Then, for instance:

Given the input function                      the operator                      returns the output function

<i>PRICE LIST</i>	
Book	List Price
Math	52.45
English	47.80
History	62.75
Biology	74.50
Poetry	64.28

**ADD-ON**<sub>SHIPPING<sub>4.25</sub></sub>

<i>NET PRICE LIST</i>	
Book	Net Price
Math	52.45 + 4.25
English	47.80 + 4.25
History	62.75 + 4.25
Biology	74.50 + 4.25
Poetry	64.28 + 4.25

In other words,

<i>PRICE LIST</i>	
Book	List Price
Math	52.45
English	47.80
History	62.75
Biology	74.50
Poetry	64.28

**ADD-ON**<sub>SHIPPING<sub>4.25</sub></sub> →

<i>NET PRICE LIST</i>	
Book	Net Price
Math	56.70
English	52.05
History	67.00
Biology	78.75
Poetry	68.53

**2.** When the functions are specified by *global input-output rules*, things are still simple.

**EXAMPLE 2.** Given the add-on function specified by the *global input-output rule*

$$x \xrightarrow{NANA_{-2.45}} NANA_{-2.45}(x) = -2.45$$

and given the input function specified by the *global input-output rule*

$$x \xrightarrow{INPUT} INPUT(x) = +13.72x$$

then the output function will be specified by the *global input-output rule*

$$\begin{aligned} x \xrightarrow{OUTPUT} OUTPUT(x) &= INPUT(x) + NANA_{-2.45}(x) \\ &= +13.72x - 2.45 \end{aligned}$$

so that, given for instance the input number  $-7.4$  and while

$$\begin{aligned} INPUT(-7.4) &= (+13.72) \cdot (-7.4) \\ &= -101.528 \end{aligned}$$

we have

$$\begin{aligned} OUTPUT(-7.4) &= (+13.72) \cdot (-7.4) - 2.45 \\ &= -103.978 \end{aligned}$$

## 8.2 Dilation Add-On Functions

We now investigate the case when the add-on function is a *dilation function* because that will very often be the case in the rest of this text.

1. When the functions are specified by *tables*, things are pretty simple.

**EXAMPLE 3.** When we shop local, there is a proportional add-on “Sales Tax”, say 7%, which we represent by the *dilation* add-on function:

$$x \xrightarrow{SALES\ TAX_{7\%}} SALES\ TAX_{7\%}(x) = 7\% \cdot x$$

Then, for instance:

given the input function

<i>PRICE LIST</i>	
Book	List Price
Math	52.45
English	47.80
History	62.75
Biology	74.50
Poetry	64.28

the operator

**ADD-ON**<sub>SALES TAX<sub>7%</sub></sub>

returns the output function

<i>NET PRICE LIST</i>	
Book	Net Price
Math	52.45 + 7% · 52.45
English	47.80 + 7% · 47.80
History	62.75 + 7% · 62.75
Biology	74.50 + 7% · 74.50
Poetry	64.28 + 7% · 64.28

In other words:

<i>PRICE LIST</i>	
Book	List Price
Math	52.45
English	47.80
History	62.75
Biology	74.50
Poetry	64.28

**ADD-ON**<sub>SALES TAX<sub>7%</sub></sub>

<i>NET PRICE LIST</i>	
Book	Net Price
Math	56.12
English	51.15
History	67.14
Biology	79.72
Poetry	68.78

2. When the functions are specified by *global input-output rules*, we have that, given the add-on function specified by the global input-output rule

$$x \xrightarrow{ADD-ON\ FUNCTION_a} ADD-ON\ FUNCTION_a(x) = ax$$

and given the input function specified by the global input-output rule

$$x \xrightarrow{INPUT} INPUT(x)$$

the output function is specified by the global input-output rule

$$\begin{aligned} x \xrightarrow{OUTPUT} OUTPUT(x) &= INPUT(x) + ADD-ON\ FUNCTION(x) \\ &= INPUT(x) + ax \end{aligned}$$

**EXAMPLE 4.** Given the add-on dilation function *SALES TAX*<sub>7%</sub> specified by the *global input-output rule*

$$x \xrightarrow{SALES\ TAX_{7\%}} SALES\ TAX_{7\%}(x) = 7\% \cdot x$$

stack pointwise

then, given the input function  $MARBLE\ LIST\ PRICE_{+4.17}$  specified by the *global input-output rule*

$$x \xrightarrow{MARBLE\ LIST\ PRICE_{+4.17}} MARBLE\ LIST\ PRICE_{+4.17}(x) = +4.17 \cdot x$$

then the output function  $MARBLE\ NET\ PRICE_{+4.17}$  will be specified by the *global input-output rule*

$$\begin{aligned} x \xrightarrow{MARBLE\ NET\ PRICE} MARBLE\ NET\ PRICE(x) &= +4.17 \cdot x + 7\% \cdot x \\ &= (+4.17 + 0.07) \cdot x \\ &= +4.24 \cdot x \end{aligned}$$

so that, given for instance the input number +7.4, while

$$\begin{aligned} MARBLE\ LIST\ PRICE_{+4.17}(+7.4) &= (+4.17) \cdot (+7.4) \\ &= +30.86 \end{aligned}$$

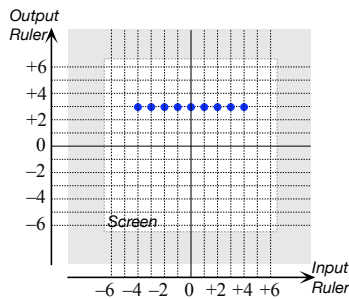
we have

$$\begin{aligned} MARBLE\ NET\ PRICE &= (+4.17) \cdot (+7.4) + (+0.07) \cdot (+7.4) \\ &= +30.86 + 0.52 \\ &= +31.38 \end{aligned}$$

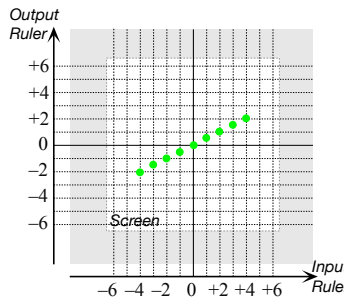
**3.** When the functions are specified by *quantitative plots*, things are a bit more difficult to *describe*: we must **stack pointwise** the plot of the add-on function on top of the plot of the input function. In other words, on each input-level line, we “count the output of the add-on function from the output of the input function”.

#### EXAMPLE 5.

Let the input function  $INPUT$  be specified by the plot:



Let the add-on function  $ADD-ON$  be specified by the plot:

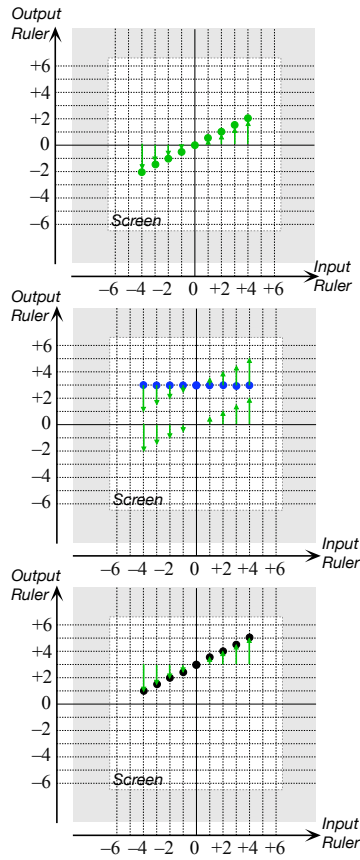


In order to get the plot for the output function  $OUTPUT$  we proceed as follows:  
For each input,

We count on the plot of *ADD-ON* the outputs it returns

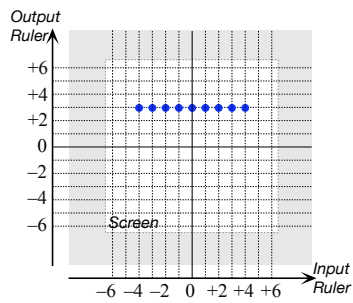
We recount these outputs on the plot of *INPUT* starting from the outputs it returns

The resulting points are the plot point of *OUTPUT*

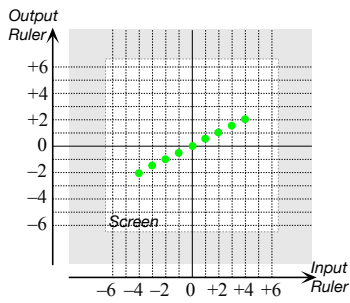


Altogether we have:

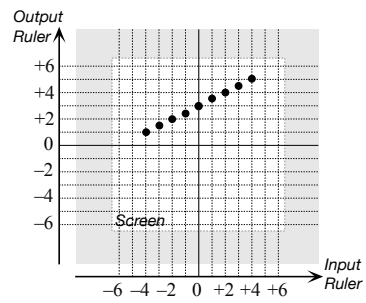
*INPUT:*



*ADD-ON:*



*OUTPUT:*



### 8.3 Power Add-On Functions

1. When the input function and the add-on function are specified by *global input-output rules*, things are fairly simple because we know how to add in algebra.

**EXAMPLE 6.** Given the add-on function *MINT* specified by the *global input-output rule*

$$x \xrightarrow{MINT} MINT(x) = -12.82x^4$$

and given the input function *TEA* specified by the *global input-output rule*

$$x \xrightarrow{TEA} TEA(x) = +49.28x^7$$

then the output function will be specified by the *global input-output rule*

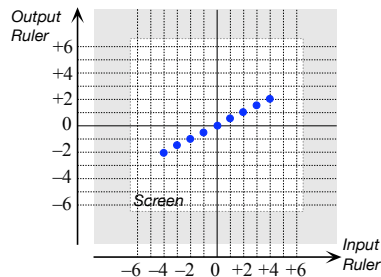
$$\begin{aligned} x \xrightarrow{OUTPUT} OUTPUT(x) &= TEA(x) + MINT(x) \\ &= +49.28x^7 - 12.82x^4 \end{aligned}$$

2. When the input function and the add-on function are specified by *quantitative plots*, we must *stack pointwise* the plot of the add-on function on top of the plot of the input function. In other words, on each input-level line, we “count the output of the add-on function from the output of the input function”.

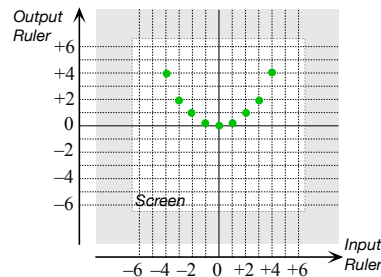
A frequent situation will be one where the input function is a *dilation function* and the add-on function is a *squaring function*.

**EXAMPLE 7.**

Let the input function *INPUT* be specified by the plot:



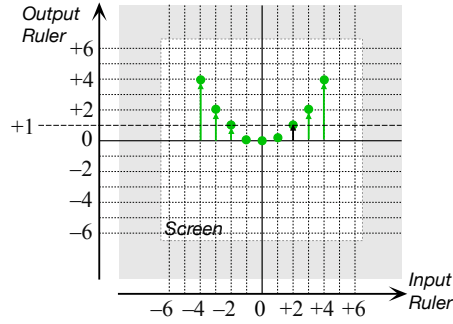
Let the add-on function *ADD-ON* be specified by the plot:



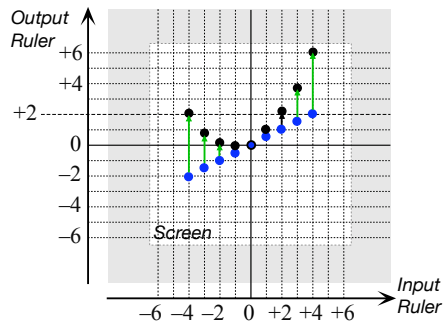
In order to get the plot for the output function *OUTPUT* we proceed as follows:

For each input,

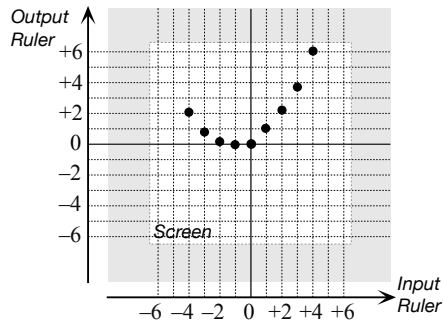
We count on the plot of *ADD-ON* the outputs it returns



We recount these outputs on the plot of *INPUT* starting from the outputs it returns

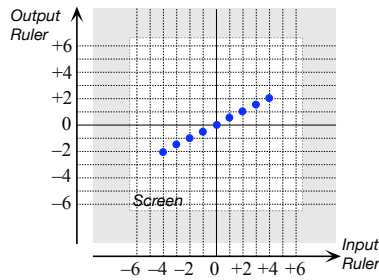


The resulting points are the plot point of *OUTPUT*

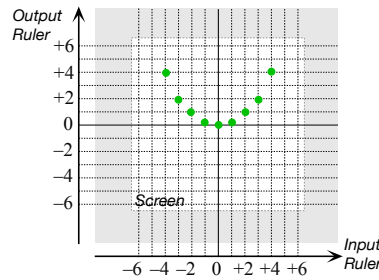


Altogether we have:

*INPUT*:



*ADD-ON*:



*OUTPUT*:

