Chapter 5

Regular Power Functions - Local Analysis

Power functions are functions that multiply or divide a given number by a number of copies of the input. We will use the following code to specify power functions:

\[(\text{coefficient})\text{base}^{\text{exponent}}\]

where

- The **coefficient** is the number being multiplied by the copies of the input. The coefficient can be any bounded number.
- The **base** is the original from which the copies are to be made. In a power function, we take the input as base.
- The **exponent** is a signed counting number that says what the function will do to the coefficient with the copies of the input.
  - The size of the exponent is the number of copies of the input to be made. (If the exponent is 0, no copy is to be made and the coefficient is to be left alone.)
  - The sign of the exponent says whether the coefficient is to be multiplied or divided by the copies of the input:
    - + says the coefficient is to be multiplied by the copies of the input,
    - − says the coefficient is to be divided by the copies of the input.
regular power function

**Example 1.**

\[ x \xrightarrow{FLIP} FLIP(x) = (+5273.1)x^{+5} \]
\[ = +5273.1 \cdot x \cdot x \cdot x \cdot x \cdot x \]

and

\[ x \xrightarrow{FLOP} FLOP(x) = (+5273.1)x^{-5} \]
\[ = +5273.1 \div x \cdot x \cdot x \cdot x \cdot x \]
\[ = +5273.1 \div x \cdot x \cdot x \cdot x \cdot x \]

but

\[ x \xrightarrow{FLAP} FLAP(x) = (+5273.1)x^{0} \]
\[ = +5273.1 \]

In this and the next chapter, though, we will deal only with **regular power functions**, that is with power functions whose exponent is different from both 0 and 1. Power functions with an exponent equal to 0 or to 1 will be called **exceptional power functions** and will be discussed in Chapter 7.

**Example 2.** The function **FLAP** specified by the global input-output rule

\[ x \xrightarrow{FLAP} FLAP(x) = (+5273.1)x^{0} \]
\[ = +5273.1 \]

is an **exceptional power function** and the function **FLIP** specified by the global input-output rule

\[ x \xrightarrow{FLIP} FLIP(x) = (+5273.1)x^{+1} \]
\[ = +5273.1 \cdot x \]
\[ = +5273.1x \]

is also an **exceptional power function** but the function **FLOP** specified by the global input-output rule

\[ x \xrightarrow{FLOP} FLOP(x) = (+5273.1)x^{-1} \]
\[ = +5273.1 \div x \]
\[ = +5273.1 \div x \]

is a **regular power function**.
5.1 Input-Output Pairs

As with any function specified by a global input-output rule, in order to get the output for a given input we must:

i. Indicate by which given input \( x \) is to be replaced

ii. Carry out the replacement of \( x \) by the given input

iii. Read the output-specifying code,

iv. Write what the output-specifying code says to do,

v. Carry out the computations.

**Example 3.** Given the function specified by the global input-output rule

\[
x \xrightarrow{FLIP} FLIP(x) = (+5 \, 273.1)x^5
\]

and given the input \(-3\), in order to find the output we proceed as follows:

i. We indicate that \( x \) is to be replaced by the given input \(-3\)

\[
\begin{array}{c|c|c|c}
  x & FLIP & FLIP(x) & (+5 \, 273.1)x^5 \\
  \hline
  x & -3 & FLIP(-3) & (+5 \, 273.1)(-3)^5 \\
\end{array}
\]

ii. We carry out the replacement of \( x \) by \(-3\) in the global input-output rule

\[
-3 \xrightarrow{FLIP} FLIP(-3) = (+5 \, 273.1)(-3)^5
\]

iii. We read the output specifying code

The output \( FLIP(-3) \) is obtained by multiplying the coefficient \(+5 \, 273.1\) by 5 copies of the given input \(-3\).

iv. We write what the output-specifying code says to do

\[
FLIP(-3) = (+5 \, 273.1) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3)
\]

v. We carry out the computations

\[
= (+5 \, 273.1) \cdot (-243)
\]

\[
= -3 \, 844 \, 089.9
\]

Depending on the circumstances, we can then write:

- From the computational viewpoint
  \[
  FLIP(-3) = -3 \, 844 \, 089.9
  \]

- From the functional viewpoint
  \[
  -3 \xrightarrow{FLIP} -3 \, 844 \, 089.9
  \]
CHAPTER 5. LOCAL ANALYSIS

features, of input-output rule

• From the graphic viewpoint
  \((-3, -3.8440899)\) is an input-output pair for the function \(FLIP\).

**Example 4.** Given the function specified by the global input-output rule

\[x \xrightarrow{FLOP} FLOP(x) = (+5.2731)x^{-5}\]

and given the input \(-3\), in order to find the output we proceed as follows:

\[
\begin{align*}
\text{i. We indicate that } x & \text{ is to be replaced by } -3 \\
FLOP & \rightarrow FLOP(x)_{|x=-3} = (+5.273.1)x_{|x=-3}^{\frac{-5}{5}}
\end{align*}
\]

\[
\begin{align*}
\text{ii. We carry out the replacement of } x & \text{ by } -3 \\
-3 & \xrightarrow{FLOP} FLOP(-3) = (+5.273.1)(-3)^{\frac{-5}{5}}
\end{align*}
\]

\[
\begin{align*}
\text{iii. We read the output specifying code:} \\
& "FLOP(-3) \text{ is obtained by dividing } +5.273.1 \text{ by 5 copies of } -3." \\
\text{iv. We write what the output-specifying code says to do} \\
& FLOP(-3) = \frac{+5.273.1}{(-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3)}^{\frac{5}{5}} \\
& = (+5.273.1) \div (-243) \\
& = -21.7
\end{align*}
\]

Depending on the circumstances, we can then write:

• From the computational viewpoint
  \(FLOP(-3) = -21.7\)

• From the functional viewpoint
  \(-3 \xrightarrow{FLOP} -21.7\)

• From the graphic viewpoint
  \((-3, -21.7)\) is an input-output pair for the function \(FLOP\).

5.2 Types of Power Functions

The features of the input-output rule of a regular power function refer to the three numbers that specify the global input-output rule. The features
of the input-output rule will correspond to the features of the graph. Since, in this text, we will mostly deal with qualitative graphs, all the features will not be equally important for us. More precisely:

1. The three features that will be important for us are:
   - **Sign exponent** which can be $+$ or $-$,
   - **Parity exponent** which can be **even** or **odd** depending on whether the size of the exponent, that is the number of copies, is **even** or **odd**.
   - **Sign coefficient** which can be $+$ or $-$.

From our point of view,
- **Size coefficient** will be irrelevant beyond the requirement in the definition of a power function that the **coefficient** be a **bounded** number.
- **Size exponent** will not be an important feature in this text because, other than being **even** or **odd**, the particular number of copies will not matter from our qualitative viewpoint.

2. Accordingly, in order to focus on the important features of regular power functions, we will often **normalize** the global input-output rule as follows:
   - We will replace the size of the coefficient by 1,
   - We will replace the size of the exponent:
     - by the word **even** when the size of the exponent is **even**
     - by the word **odd** when the size of the exponent is **odd**

**Example 5.** The function specified by the global input-output rule

\[
    x \xrightarrow{BLIP} BLIP(x) = (-160.42)x^{7}
\]

is a power function whose **global input-output rule** has the following features
- **Sign exponent** $BLIP = +$,
- **Parity exponent** $BLIP = odd$.
- **Sign coefficient** $BLIP = -$.

and which we can normalize as

\[
    x \xrightarrow{BLIP} BLIP(x) = (-1)x^{1 + odd}
\]

**Example 6.** The function specified by the global input-output rule

\[
    x \xrightarrow{MILK} MILK(x) = (+4518.32)x^{6}
\]

is a power function whose **global input-output rule** has the following features
• Sign exponent MILK = \(-\),
• Parity exponent MILK = even,
• Sign coefficient MILK = +,

and which we can normalize as

\[ x \xrightarrow{\text{MILK}} \text{MILK}(x) = (+1)x^{-\text{even}} \]

3. From the qualitative viewpoint that we will be taking, there will therefore be eight types of regular power functions:

<table>
<thead>
<tr>
<th>Sign exponent</th>
<th>Parity exponent</th>
<th>Sign coefficient</th>
<th>Input-output rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>even</td>
<td>+</td>
<td>((+1)x^{+ \text{even}})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>−</td>
<td>((-1)x^{+ \text{even}})</td>
</tr>
<tr>
<td></td>
<td>odd</td>
<td>+</td>
<td>((+1)x^{+ \text{odd}})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>−</td>
<td>((-1)x^{+ \text{odd}})</td>
</tr>
<tr>
<td>−</td>
<td>even</td>
<td>+</td>
<td>((+1)x^{- \text{even}})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>−</td>
<td>((-1)x^{- \text{even}})</td>
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<tr>
<td></td>
<td>odd</td>
<td>+</td>
<td>((+1)x^{- \text{odd}})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>−</td>
<td>((-1)x^{- \text{odd}})</td>
</tr>
</tbody>
</table>

**LOCAL ANALYSIS NEAR \(\infty\)**

The computations for getting the place of the local graph near \(\infty\) will use the following from Chapter 2:
- Rule for Sign Multiplication and Division
- Definitions of large and small.
- Size Multiplication Theorem and Size Division Theorem

**5.3 Place of the Local Graph Near \(+\infty\)**

Since the local graph near \(+\infty\) is for \(+\)large inputs, the local graph near \(+\infty\) will be somewhere in the following area:
so that the local graph near \(+\infty\) will be offscreen regardless of the size of the outputs.

1. More precisely, since the inputs are positive large,

i. We get the sign of the output from the sign of the coefficient since the inputs are positive and, by the Rule for Sign Multiplication, any number of copies of + will multiply to +. So, depending on the sign of the coefficient, the local graph near +\(\infty\) will be either one of the following:

![Positive coefficient](image1)

or

![Negative coefficient](image2)

ii. We get the size of the output from the sign of the exponent:

- If the exponent is positive, the coefficient will be multiplied by copies of large so that, by the Size Multiplication Theorem, the output will be large and the place of the local graph near +\(\infty\) will be, depending on the sign of the coefficient, either one of the following:

![Positive exponent](image3)

or

![Negative exponent](image4)

- If the exponent is negative, the coefficient will be divided by copies of large so that, by the Size Division Theorem, the output will be small and the place of the local graph near +\(\infty\) will be, depending on the sign of the coefficient, either one of the following:
2. In practice, though, we will deal at the same time with both the size and the sign of the inputs.

**Example 7.** Given the function specified by the global input-output rule

\[ x \xrightarrow{\text{JADE}} \text{JADE}(x) = (-83.91)x^5 \]

in order to find the place of the local graph near \(+\infty\):

i. We normalize the global input output rule:

\[ x \xrightarrow{\text{JADE}} \text{JADE}(x) = (-1)x^{+\text{odd}} \]

ii. We compute the output for inputs that are \(+\text{large}\):

\[
\begin{align*}
x \xrightarrow{\text{JADE}} \text{JADE}(x) \bigg|_{x=+\text{large}} &= (-1)x^{+\text{odd}} \bigg|_{x=+\text{large}} \\
&= (-1)(+\text{large})^{+\text{odd}} \\
&= (-1) \cdot (+) \cdot \ldots \cdot (+) \cdot (\text{large}) \cdot \ldots \cdot (\text{large}) \\
&= (-1) \cdot (+) \cdot \ldots \cdot (+) \cdot (\text{large}) \cdot \ldots \cdot (\text{large}) \\
&= -\text{large}
\end{align*}
\]
iii. And so we have that:
$JADE(+\text{large}) = -\text{large}$
and that the place of the local graph of $JADE$ near $+\infty$ is:

![Diagram of graph place near $+\infty$]

**Example 8.** Given the function specified by the global input-output rule

$$x \xrightarrow{NADE} NADE(x) = (-18.56)x^{-5}$$

in order to find the local place of the local graph near $+\infty$:

i. We normalize the global input output rule:

$$x \xrightarrow{NADE} NADE(x) = (-1)x^{-\text{odd}}$$

ii. We compute the output for inputs that are $+\text{large}$:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\xrightarrow{NADE} NADE(x)$</th>
<th>$x \leftarrow +\text{large}$</th>
<th>$(-1)x^{-\text{odd}}$</th>
<th>$(-1)(+\text{large})^{-\text{odd}}$</th>
<th>$(-1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$+\text{large}$</td>
<td></td>
</tr>
<tr>
<td>$+$</td>
<td>$+\text{large}$</td>
<td>$+\text{large}$</td>
<td>$-1$</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

and since, by the Rule for Sign Multiplication, any number of copies of $+$ multiply to $+$

$$= -1 \cdot (+\text{large}) \cdot \ldots \cdot (+\text{large})$$

and since, by the Definition of $\text{large}$, any number of copies of $\text{large}$ multiply to $\text{large}$

$$= -1 \cdot (\text{large})$$
and since, by the **Size Division Theorem**, bounded divided by large is small

\[ \text{NADE}(\text{+large}) = -\text{small} \]

iii. And so we have that:

\[ \text{NADE}(\text{+large}) = -\text{small} \]

and that the place of the local graph of DATE near \(+\infty\) is:

---

### 5.4 Place of the Local Graph Near \(-\infty\)

Since the local graph near \(-\infty\) is for \(-\text{large}\) inputs, the local graph near \(-\infty\) will be somewhere in the following area:

that is the local graph near \(-\infty\) will be offscreen regardless of the size of the outputs.

1. If all we want is the local graph near \(-\infty\), we proceed exactly in the same manner as for the local graph near \(+\infty\).

**Example 9.** Given the function specified by the global input-output rule

\[ x \xrightarrow{KATE} KATE(x) = (-13.14) \cdot x^{-24} \]

in order to find the place of the *local graph* of KATE near \(-\infty\):

i. We *normalize* the global input output rule:

\[ x \xrightarrow{KATE} KATE(x) = (-1) \cdot x^{\text{even}} \]

ii. We *compute* the output for inputs that are \(-\text{large}\):
iii. And so we have that:
\[ KATE(-\text{large}) = -\text{small} \]
and that the place of the local graph of KATE near \(-\infty\) is:

2. If we want the local graph near \(-\infty\) as part of the local graph near \(\infty\), after we have gotten the local graph near \(+\infty\) we obtain the local graph near \(-\infty\) from the local graph near \(+\infty\) by the following

**THEOREM 1 (Place Near \(-\infty\)).** For a regular power function, the local place near \(-\infty\) is obtained by flipping the local place near \(+\infty\) according to the parity of the exponent:

- When the exponent is even, the local place near \(-\infty\) is obtained by flipping the local graph near \(+\infty\) horizontally:
EXAMPLE 10. Given the function specified by the global input-output rule

\[ x \xrightarrow{DAVE} DAVE(x) = (-83.17)x^{-13} \]

in order to find the place of the local graph of \( DAVE \) near \( \infty \)

i. We get the place of the local graph near \( +\infty \) by computing from the global input-output rule as we did above:

ii. We get the place of the local graph near \( -\infty \), by flipping the local graph near \( +\infty \) according to the Place Near \( -\infty \) Theorem:
iii. Altogether then, the place of the local graph near $\infty$ is:

3. The case for the **Place Near $-\infty$ Theorem** is entirely based on the **Rule for Signs Multiplication**:

- When Parity exponent = even, multiplying the signs of the copies of the input will give $+$. The sign of the output will then be the sign of the coefficient multiplied by $+$ and thus will be **the same as** the sign of the coefficient. So the place near $-\infty$ will be on the same side of 0 as the place near $+\infty$,

- When Parity exponent = odd, multiplying the signs of the copies of the input will give $-$. The sign of the output will then be the sign of the coefficient multiplied by $+$ and thus will be **the opposite of** the sign of the coefficient. So the place near $-\infty$ will be on the other side of 0 from the place near $+\infty$.

5.5 **Shape of the Local Graph Near $\infty$**

The shape of the local graph of a regular power function near $\infty$ is **forced** by its place.

1. More precisely:

**THEOREM 2 (Shape Near $\infty$).** As inputs get nearer to $\infty$:

- Positive exponents power functions “**flatline**” vertically:

- Negative exponents power functions “**flatline**” horizontally:
2. Making the case for the Shape Near $\infty$ Theorem computationally is not difficult but is a bit longish so we will not do it here. On the other hand, it is easy to see that the shape of the local graph near $\infty$ could not be any other than what the Shape Near $\infty$ Theorem says.

**Example 11.** Given the place

i. The slope cannot be $\backslash$ as in

because, as inputs get larger, outputs would get smaller while the definition of large says that outputs have to get larger. So, the slope has to be $/$.

ii. The concavity cannot be $\cap$ as in

because, as inputs get larger, outputs would eventually cease to get larger. So, the concavity has to be $\cup$.

Altogether then, since the slope has to be $/$ and the concavity has to be $\cup$, the shape of the local graph near $+\infty$ can only be:

3. In practice, to get the local graph near $\infty$ we:
5.5. GRAPH SHAPE NEAR $\infty$

i. Compute the place of the local graph near $+\infty$.

ii. Get the place of the graph near $-\infty$ by flipping the place of the graph near $+\infty$ according to the **Place Near $-\infty$ Theorem**.

iii. Get the shape of the graph near $\infty$ by the **Shape Near $\infty$ Theorem**.

**Example 12.** Given the function specified by the global input-output rule

$$x \xrightarrow{KATE} KATE(x) = (-13.14) \cdot x^{24}$$

in order to find the local graph of $KATE$ near $\infty$.

i. We get the place of the local graph near $+\infty$ from the global input-output rule by computing as we did above:

ii. We get the place of the local graph near $-\infty$, by flipping the local graph near $+\infty$ according to the **Place Near $-\infty$ Theorem**:

iii. Altogether then, the place of the local graph near $\infty$ is:
iv. We get the shape of the local graph near $\infty$ by the Shape Near $\infty$ Theorem

LOCAL ANALYSIS NEAR 0

The computations for getting the place of the local graph near $\infty$ will use the following from Chapter 2:

- Rule for Sign Multiplication and Division
- Definition of large and small.
- Size Multiplication Theorem and Size Division Theorem as a result of which we immediately have the extremely important

THEOREM 3 ($\infty$-Height). For positive-exponent power functions, all bounded inputs have bounded height and 0 is a 0-height input.
For negative-exponent power functions, 0 is an $\infty$-height input, that is nearly inputs have $\infty$ height.

EXAMPLE 13. Given the function $RHON$ specified by the global input-output rule

$$x \xrightarrow{RHON} RHON(x) = (+93.29) \cdot x^{+3}$$

we have:

$$\text{bounded } \xrightarrow{RHON} \text{RHON(bounded)} = (+93.29) \cdot \text{bounded}^{+3}$$
$$= (+93.29) \cdot \text{bounded} \cdot \text{bounded} \cdot \text{bounded}$$

and, by the Size Multiplication Theorem

$$= \text{bounded}$$

In particular,

$$\text{small } \xrightarrow{RHON} \text{RHON(small)} = (+93.29) \cdot \text{small}^{+3}$$
$$= (+93.29) \cdot \text{small} \cdot \text{small} \cdot \text{small}$$

and, by the Size Multiplication Theorem

$$= \text{small}$$

so that:

$$0 \xrightarrow{RHON} RHON(0) = 0$$
Example 14. Given the function $DION$ specified by the global input-output rule

$$x \xrightarrow{DION} DION(x) = (+13.72) \cdot x^{-4}$$

we have:

$$bounded \xrightarrow{DION} DION(bounded) = (+13.72) \cdot bounded^{-4}$$

$$= \frac{+13.72}{bounded \cdot bounded \cdot bounded \cdot bounded}$$

and, by the Size Division Theorem

$$= bounded$$

However,

$$small \xrightarrow{DION} DION(small) = (+13.72) \cdot small^{-4}$$

$$= \frac{+13.72}{small \cdot small \cdot small \cdot small}$$

and, by the Size Division Theorem

$$= large$$

so that:

$$0 \xrightarrow{DION} DION(0) = \infty$$

5.6 Place of the Local Graph Near $0^+$

Since the local graph near $0^+$ is for $+small$ inputs, the local graph near $0^+$ will be somewhere in the following area

![Local Graph Area Diagram](image)

so that is the local graph near $0^+$ will be onscreen or offscreen depending on the size of the outputs.
1. More precisely, since the inputs are positive small,

i. We get the sign of the output from the sign of the coefficient since the inputs are positive and, by the Rule for Sign Multiplication, any number of copies of + will multiply to +. So the local graph near 0⁺ will be, depending on the sign of the coefficient, either one of the following:

ii. We get the size of the output from the sign of the exponent:

- If the exponent is positive, the coefficient will be multiplied by the copies of the small input so that, by the Size Multiplication Theorem, the output will be small and the place of the local graph near 0⁺ will be, depending on the sign of the coefficient, either one of the following:

- If the exponent is negative, the coefficient will be divided by the copies of the small input so that, by the Size Division Theorem, the output will be large and the place of the local graph near +∞ will be, depending on the sign of the coefficient, either one of the following:
5.6. **GRAPH PLACE NEAR 0⁺**

2. In practice, though, we will deal at the same time with both the **size** and the **sign** of the inputs.

**Example 15.** Given the function specified by the *global input-output rule*

\[
x \xrightarrow{\text{KATE}} \text{KATE}(x) = (-13.14) \cdot x^{+24}\]

in order to find the **place** of the local graph near \(0⁺\):

**i.** We *normalize* the global input output rule:

\[
x \xrightarrow{\text{KATE}} \text{KATE}(x) = (-1) \cdot x^{+\text{even}}\]

**ii.** We *compute* the output for inputs that are \(+\text{small}\):

\[
x \xrightarrow{\text{KATE}} \text{KATE}(x) \bigg|_{x \leftarrow +\text{small}} = (-1) \cdot (+) \cdot (+) \cdots \cdot (+) \cdot \text{even number of copies of } +\text{small}\]

and since by the **Rule for Sign Multiplication**, *any* number of copies of \(+\) multiply to \(+\)

\[
= (-1) \cdot (+) \cdot \text{even number of copies of } +\text{small}\]

and since, by the **Definition of small**, *any* number of copies of \(\text{small}\) multiply to \(\text{small}\)

\[
= (-1) \cdot ((+)) \cdot \text{even number of copies of } \text{small}\]
and since, by the Size Multiplication Theorem, bounded multiplied by small is small

\[ = -\text{small} \]

iii. And so we have that:

\[ KATE(\text{+small}) = -\text{small} \]

and that the place of the local graph of \( KATE \) near \( 0^+ \) is:

**Example 16.** Given the function specified by the global input-output rule

\[ x \xrightarrow{\text{DATE}} \text{DATE}(x) = (-13.14) \cdot x^{-8} \]

in order to find the place of the local graph near \( 0^+ \):

i. We normalize the global input output rule:

\[ x \xrightarrow{\text{DATE}} \text{DATE}(x) = (-1) \cdot x^{-\text{even}} \]

ii. We compute the output for inputs that are +small:

\[
\begin{align*}
\left. x \xrightarrow{\text{DATE}} \text{DATE}(x) \right|_{x = +\text{small}} &= (-1)x^{-\text{even}} \bigg|_{x = +\text{small}} \\
&= (-1) \cdot (+\text{small})^{-\text{even}} \\
&= (-1) \cdot (-1)^{\text{even number of copies of } +\text{small}} \\
&= (-1)^{\text{even number of copies of } +} \cdot (\text{small})^{\text{even number of copies of } \text{small}} \\
&= -1
\end{align*}
\]

and since, by the Rule for Sign Multiplication, any number of copies of + multiply to +

\[
(+)^{\text{even number of copies of } +} \cdot (\text{small})^{\text{even number of copies of } \text{small}} = -1
\]

and since, by the Definition of small, any number of copies of small multiply to small

\[
(-)^{\text{odd number of copies of } \text{small}} = -1
\]
and since, by the **Size Division Theorem**, \( \text{bounded divided by small is small} \)
\[ = \text{large} \]

iii. And so we have that:
\[ \text{DATE}(+\text{small}) = \text{large} \]
and that the **place** of the local graph of \( \text{DATE} \) near \( 0^+ \) is:

5.7 **Place of the Local Graph Near 0−**

Since the local graph near \( 0^- \) is for \( -\text{small} \) inputs, the local graph near \( 0^- \) will be somewhere in the following area that is the local graph near \( 0^- \) will be onscreen or offscreen **depending** on the size of the **outputs**.

1. If all we want is the local graph near \( 0^- \), we proceed exactly in the same manner as for the local graph near \( 0^+ \).

**EXAMPLE 17.** Given the function specified by the **global input-output rule**

\[ x \xrightarrow{\text{DATE}} \text{DATE}(x) = (-13.14) \cdot x^{-24} \]

in order to find the **place** of the local graph near \( 0^- \):

i. We **normalize** the global input output rule:
\[ x \xrightarrow{\text{DATE}} \text{DATE}(x) = (-1) \cdot x^{\text{even}} \]

ii. We **compute** the output for inputs that are \( -\text{small} \):

\[
\begin{align*}
x \bigg|_{x \leftarrow -\text{small}} \xrightarrow{\text{DATE}} \text{DATE}(x) \bigg|_{x \leftarrow -\text{small}} &= (-1)x^{\text{even}} \\
&= (-1) \cdot (-\text{small})^{-\text{even}}
\end{align*}
\]
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\[
\frac{-1}{(-\text{small}) \cdot \ldots \cdot (-\text{small})} = \frac{-1}{\left(\frac{-1}{\text{small}}\right) \cdot \ldots \cdot \left(\frac{-1}{\text{small}}\right)}
\]

and since, by the **Rule for Sign Multiplication**, an even number of copies of \(-\) multiplies to \(+\)

\[
\frac{-1}{\left(\frac{1}{\text{small}}\right) \cdot \ldots \cdot \left(\frac{1}{\text{small}}\right)}
\]

and since, by the **Definition of small**, copies of \(\text{small}\) multiply to \(\text{small}\)

\[
\frac{-1}{\left(\frac{1}{\text{small}}\right) \cdot \left(\frac{1}{\text{small}}\right)}
\]

and since, by the **Size Division Theorem**, bounded divided by \(\text{small}\) is large

\[
-\text{large}
\]

### iii. And so we have that:

\(DATE(-\text{small}) = -\text{large}\)

and that the place of the local graph of \(DATE\) near \(0^-\) is:

<table>
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<tr>
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<td>0</td>
</tr>
<tr>
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<td>(\infty)</td>
</tr>
</tbody>
</table>

#### 2. If we want the local graph near \(0^-\) as part of the local graph near \(0\), after we have gotten the local graph near \(+\infty\) we obtain the local graph near \(0^-\) from the local graph near \(0^+\) by the following

**THEOREM 4 (Place Near \(0^-\)).** *For a regular power function, the local place near \(0^-\) is obtained by flipping the local place near \(0^+\) according to the parity of the exponent:*

- When the exponent is even, the local place near \(0^-\) is obtained by flipping the local graph near \(0^+\) horizontally:
5.7. GRAPH PLACE NEAR $0^-$

- When the exponent is odd, the local place near $0^-$ is obtained by flipping the local graph near $0^+$ diagonally:

**Example 18.** Given the function specified by the global input-output rule

$$x \xrightarrow{DATE} DATE(x) = (-13.14) \cdot x^{-24}$$

in order to find the place of the local graph of $DATE$ near 0

**i.** We get the place of the local graph near $0^+$ by computing from the global input-output rule as we did above:

**ii.** We get the place of the local graph near $0^-$, by flipping the local graph near $0^+$ according to the **Place Near $0^-$ Theorem**:
iii. Altogether then, the place of the local graph near 0 is:

3. The case for the Place Near 0⁻ Theorem goes exactly the same way as the case for the Place Near −∞ Theorem that is it is entirely based on the Rule for Signs Multiplication:

- When Parity exponent = even, multiplying the signs of the copies of the input will give +. The sign of the output will then be the sign of the coefficient multiplied by + and thus will be the same as the sign of the coefficient. So the place near −∞ will be on the same side of 0 as the place near +∞,
- When Parity exponent = odd, multiplying the signs of the copies of the input will give −. The sign of the output will then be the sign of the coefficient multiplied by + and thus will be the opposite of the sign of the coefficient. So the place near −∞ will be on the other side of 0 from the place near +∞.

5.8 Shape of the Local Graph Near 0

The shape of the local graph of a regular power function near 0 is forced by its place.

1. More precisely:

THEOREM 5 (Shape Near 0). As inputs get nearer to 0:

- Positive exponents power functions “flatline” horizontally:

- Negative exponents power functions “flatline” vertically:
2. Making the case for the **Shape Near 0 Theorem** computationally is not difficult but is a bit longish so we will not do it here. On the other hand, it is easy to see that the shape of the local graph near 0 could not be any other than what the **Shape Near 0 Theorem** says.

**Example 19.** Given the place

i. The **slope** cannot be \( \searrow \) as in

because, as inputs get smaller, outputs would get smaller while the global input-output rule says that outputs have to get **larger**.

So, the slope has to be \( \nearrow \).

ii. The **concavity** cannot be \( \nearrow \) as in

because, as inputs get smaller, outputs would eventually cease to get **larger**.

So, the concavity has to be \( \searrow \).

Altogether then, since the slope has to be \( \nearrow \) and the concavity has to be \( \searrow \), the shape of the local graph near \( 0^+ \) can only be:
3. In practice, to get the local graph near 0 we:

i. **Compute** from the global input-output rule the *place* of the local graph near $0^+$.

ii. Get the *place* of the graph near $0^-$ by *flipping* the place of the graph near $0^+$ according to the **Place Near $0^-$ Theorem**.

iii. Get the *shape* of the graph near 0 by the **Shape Near 0 Theorem**.

In other words, to get the local graph near 0, we proceed as just as we did to get the local graph near $\infty$.

**Example 20.** Given the function specified by the global input-output rule

$$x \xrightarrow{KATE} KATE(x) = (-13.14) \cdot x^{+24}$$

in order to find the local graph of $KATE$ near 0.

i. We get the *place* of the local graph near $0^+$ from the global input-output rule by computing as we did above:

ii. We get the *place* of the local graph near $0^-$, by flipping the local graph near $0^+$ according to the **Place Near $0^-$ Theorem**:

iii. Altogether then, the *place* of the local graph near 0 is:
5.8. GRAPH SHAPE NEAR 0

iv. We get the shape of the local graph near 0 by the Shape Near 0 Theorem

EXAMPLE 21. Given the function specified by the global input-output rule

\[ x \xrightarrow{DAVE} DAVE(x) = (+83.17)x^{+13} \]

in order to find the local graph of DAVE near 0.

i. We get the place of the local graph near 0+ by computing from the global input-output rule as we did above:

ii. We get the place of the local graph near 0−, by flipping the local graph near 0+ according to the Place Near 0− Theorem:

iii. Altogether then, the place of the local graph near 0 is:
iv. We get the shape of the local graph near 0 by the Shape Near 0 Theorem.
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