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add-on function
row-wise

Chapter 8

Add-On Operators

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Operators are “super-functions” in the sense that they do with functions what functions do with numbers. More precisely, given an **input function**, an operator returns an **output function**. So, if *INPUT* is the input function and if *OUTPUT* if the function that the operator outputs, then we have

$$\underbrace{\left[x \xrightarrow{INPUT} INPUT(x) \right]}_{\text{Input function}} \xrightarrow{\text{OPERATOR}} \underbrace{\left[x \xrightarrow{OUTPUT} OUTPUT(x) \right]}_{\text{Output function}}$$

In this text, though, we will use only **add-on operators**, that is operators that add to $INPUT(x)$, the output of the input function *INPUT*, the output of a given function, referred to as **add-on function**. In other words, if *ADD-ON* is the given add-on function, then, for each input x , we have:

$$OUTPUT(x) = INPUT(x) + ADD-ON(x)$$

8.1 Functions Specified by Input-Output Tables

Even though later we will not deal with functions specified by *input-output tables*, here we will do so in order to illustrate the way add-on operators work because, when the functions are specified by *input-output tables*, things are pretty simple as we add-on **row-wise**.

1. The simplest add-on operators are those where the add-on function is a *constant function* so that, for any input, the output function returns the output of the input function plus the *constant output* of the add-on function.

EXAMPLE 1. When we shop online, a fixed *shipping charge* is usually added-on to the *list price* to get the *pay price*. For instance, in the case of textbooks,

Given the function the **add-on operator** returns the function

<i>LIST PRICES</i>		<i>SHIPPING</i>	<i>PAY PRICES</i>		
x	$LIST(x)$	x	$SHIP(x)$	x	$PAY(x)$
Math	52.45	Math	3.45	Math	55.90
English	47.80	English	3.45	English	51.25
History	62.75	History	3.45	History	66.20
Biology	74.50	Biology	3.45	Biology	77.95
Poetry	64.25	Poetry	3.45	Poetry	67.70

Input function → Output function

2. The next simplest add-on operators are those where the add-on function is a *dilation function* so that, for each input, the output function returns the output of the input function plus the dilation coefficient of the add-on function multiplied by the output of the input function.

EXAMPLE 2. When we shop local, a *sales tax* is usually added-on to the *list price* to get the *pay price*. For instance, in the case of textbooks,

Given the function the **add-on operator** : returns the function:

<i>LIST PRICES</i>		<i>SALES TAX</i>	<i>PAY PRICES</i>		
x	$LIST(x)$	x	$TAX(x)$	x	$PAY(x)$
Math	52.45	Math	7% · 52.45	Math	56.12
English	47.80	English	7% · 47.80	English	51.15
History	62.75	History	7% · 62.75	History	67.14
Biology	74.50	Biology	7% · 74.50	Biology	79.72
Poetry	64.25	Poetry	7% · 64.25	Poetry	68.75

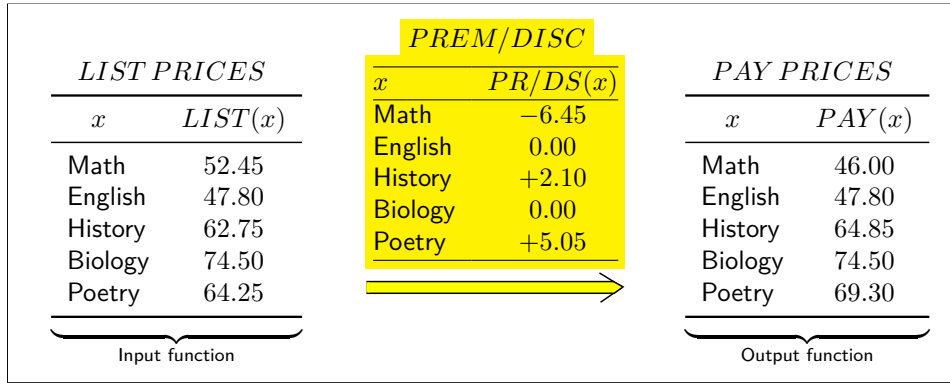
Input function → Output function

3. The add-on function, though, need not be as simple as in the above two cases.

EXAMPLE 3. Depending on how popular a product is, a *premium* or a *discount* may be added-on to the *list price* to get the *pay price*. For instance, in the case of textbooks,

raise/lower
tilt

Given the function: the **add-on operator** : returns the function:



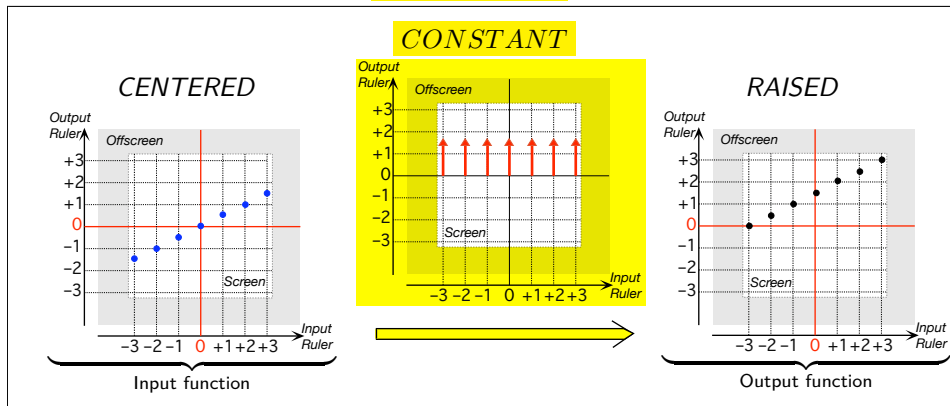
8.2 Functions Specified by Plots

Again, even though we will not deal later with functions specified by *plots*, here we will do so in order to illustrate the way add-on operators work when, as will be the case later on, functions are specified by *graphs*.

1. A frequent case will be when the add-on operator is specified by a *constant function*. The effect of such an add-on operator is to **raise/lower** the input function.

EXAMPLE 4.

Given the function: the **add-on operator** : returns the function:

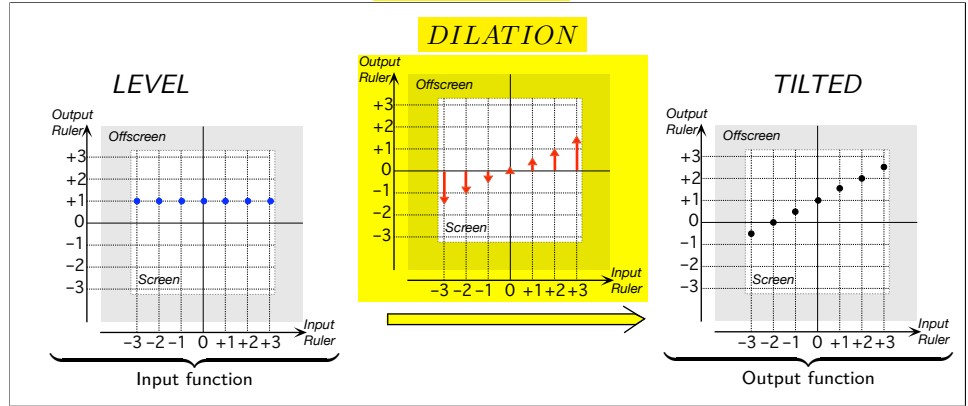


2. Another frequent case will be when the add-on operator is specified by a *dilation function*. The effect of such an add-on operator is to **tilt** the input function.

bend

EXAMPLE 5.

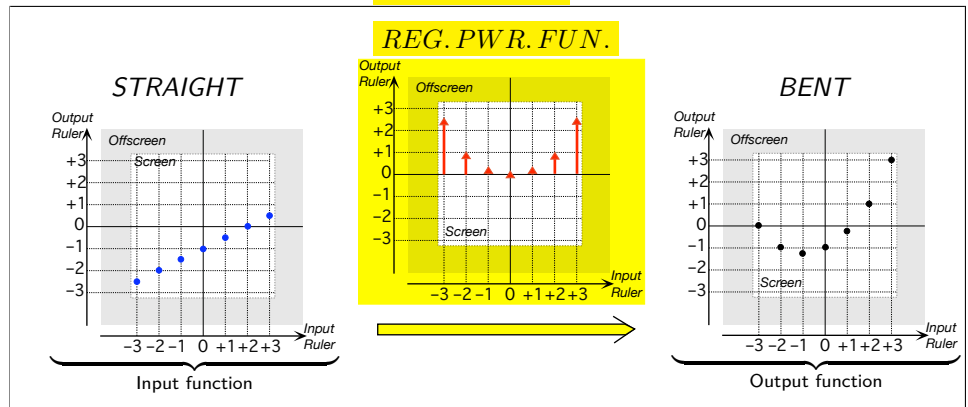
Given the function: the **add-on operator** : returns the function:



3. Finally, yet another frequent case will be when the add-on operator is specified by a *regular power function*. The effect of such an add-on operator is to **bend** the input function.

EXAMPLE 6.

Given the function: the **add-on operator** : returns the function:



8.3 Functions Specified by Input-Output Rules

=====OK SO FAR=====

When the functions are specified by *global input-output rules*, things are

still fairly simple:

Given the *input function* specified by the *global input-output rule*

$$x \xrightarrow{INPUT} INPUT(x) = \text{phrase specifying the output of } INPUT$$

the *add-on operator* specified by the *add-on function* specified by the *global input-output rule*

$$x \xrightarrow{ADD-ON} ADD-ON(x) = \text{phrase specifying the output of } ADD-ON$$

returns the *output function* specified by the *global input-output rule*

$$x \xrightarrow{OUTPUT} OUTPUT(x) = INPUT(x) + ADD-ON(x)$$

$$= \text{phrase specifying the output of } INPUT$$

$$+ \text{phrase specifying the output of } ADD-ON$$

1. When the add-on function f is a *constant function* specified by the global input-output rule

$$x \xrightarrow{f} f(x) = a$$

where a is the *constant coefficient*, the *OUTPUT* function that the add-on operator returns for the *INPUT* function is the function specified by the global input-output rule:

$$x \xrightarrow{OUTPUT} OUTPUT(x) = INPUT(x) + a$$

2. When the add-on function f is a *dilation function* specified by the global input-output rule

$$x \xrightarrow{f} f(x) = ax$$

where a is the *dilation coefficient*, the *OUTPUT* function that the add-on operator returns for the *INPUT* function is the function specified by the global input-output rule:

$$x \xrightarrow{OUTPUT} OUTPUT(x) = INPUT(x) + ax$$

1.

2. EXAMPLE 7. Given the (constant) add-on function specified by the *global input-output rule*

$$x \xrightarrow{ADD-ON_{-2.45}} ADD-ON_{-2.45}(x) = -2.45$$

then, given the *input function* specified by the *global input-output rule*

$$x \xrightarrow{INPUT} INPUT(x) = +13.72x$$

the output function will be specified by the *global input-output rule*

$$\begin{aligned} x \xrightarrow{OUTPUT} OUTPUT(x) &= INPUT(x) + ADD-ON_{-2.45}(x) \\ &= +13.72x - 2.45 \end{aligned}$$

so that given, say, the input number -7.4 we have

$$\begin{aligned} INPUT(-7.4) &= (+13.72) \cdot (-7.4) \\ &= -101.528 \end{aligned}$$

and

$$\begin{aligned} OUTPUT(-7.4) &= (+13.72) \cdot (-7.4) - 2.45 \\ &= -103.978 \end{aligned}$$

8.4 Dilation Add-On Functions

We now investigate the case when the add-on function is a *dilation function* because that will very often be the case in the rest of this text.

1. When the functions are specified by *tables*, things are pretty simple.

2. When the functions are specified by *global input-output rules*, we have that, given the add-on function specified by the global input-output rule

$$x \xrightarrow{ADD-ON FUNCTION_a} ADD-ON FUNCTION_a(x) = ax$$

and given the input function specified by the global input-output rule

$$x \xrightarrow{INPUT} INPUT(x)$$

the output function is specified by the global input-output rule

$$\begin{aligned} x \xrightarrow{OUTPUT} OUTPUT(x) &= INPUT(x) + ADD-ON FUNCTION(x) \\ &= INPUT(x) + ax \end{aligned}$$

EXAMPLE 8. Given the add-on dilation function $SALES TAX_{7\%}$ specified by the *global input-output rule*

$$x \xrightarrow{SALES TAX_{7\%}} SALES TAX_{7\%}(x) = 7\% \cdot x$$

then, given the input function $MARBLE\ LIST\ PRICE_{+4.17}$ specified by the *global input-output rule*

$$x \xrightarrow{MARBLE\ LIST\ PRICE_{+4.17}} MARBLE\ LIST\ PRICE_{+4.17}(x) = +4.17 \cdot x$$

then the output function $MARBLE\ NET\ PRICE_{+4.17}$ will be specified by the *global input-output rule*

$$\begin{aligned} x \xrightarrow{MARBLE\ NET\ PRICE} MARBLE\ NET\ PRICE(x) &= +4.17 \cdot x + 7\% \cdot x \\ &= (+4.17 + 0.07) \cdot x \\ &= +4.24 \cdot x \end{aligned}$$

so that, given for instance the input number +7.4, while

$$\begin{aligned} MARBLE\ LIST\ PRICE_{+4.17}(+7.4) &= (+4.17) \cdot (+7.4) \\ &= +30.86 \end{aligned}$$

we have

$$\begin{aligned} MARBLE\ NET\ PRICE &= (+4.17) \cdot (+7.4) + (+0.07) \cdot (+7.4) \\ &= +30.86 + 0.52 \\ &= +31.38 \end{aligned}$$

3.

8.5 Power Add-On Functions

1. When the input function and the add-on function are specified by *global input-output rules*, things are fairly simple because we know how to add in algebra.

EXAMPLE 9. Given the add-on function $MINT$ specified by the *global input-output rule*

$$x \xrightarrow{MINT} MINT(x) = -12.82x^{+4}$$

and given the input function TEA specified by the *global input-output rule*

$$x \xrightarrow{TEA} TEA(x) = +49.28x^{+7}$$

then the output function will be specified by the *global input-output rule*

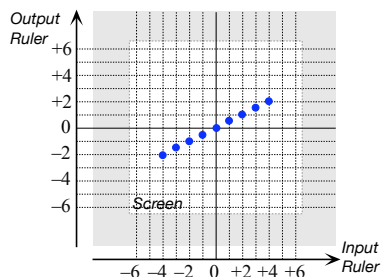
$$\begin{aligned} x \xrightarrow{OUTPUT} OUTPUT(x) &= TEA(x) + MINT(x) \\ &= +49.28x^{+7} - 12.82x^{+4} \end{aligned}$$

2. When the input function and the add-on function are specified by *quantitative plots*, we must *stack pointwise* the plot of the add-on function on top of the plot of the input function. In other words, on each input-level line, we “count the output of the add-on function from the output of the input function”.

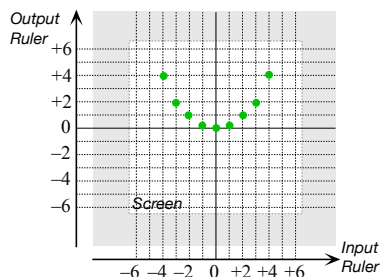
A frequent situation will be one where the input function is a *dilation function* and the add-on function is a *squaring function*.

EXAMPLE 10.

Let the input function *INPUT* be specified by the plot:



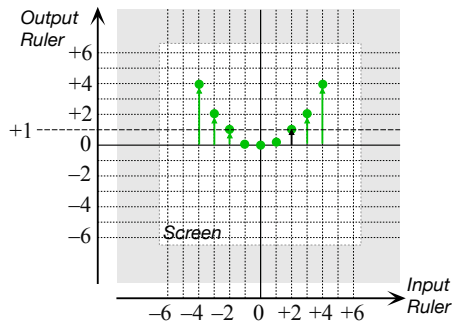
Let the add-on function *ADD-ON* be specified by the plot:



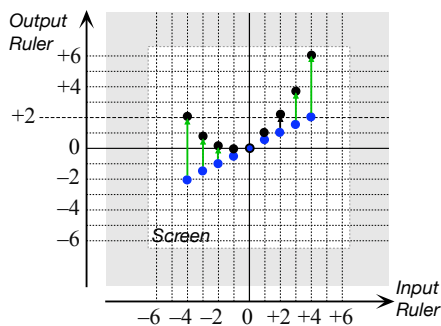
In order to get the plot for the output function *OUTPUT* we proceed as follows:

For each input,

We count on the plot of *ADD-ON* the outputs it returns

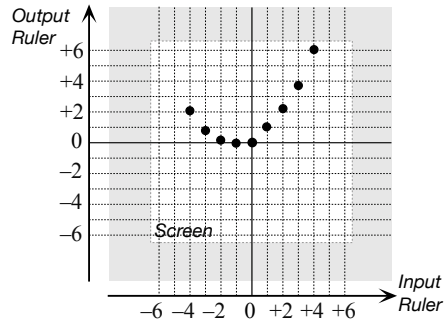


We recount these outputs on the plot of *INPUT* starting from the outputs it returns



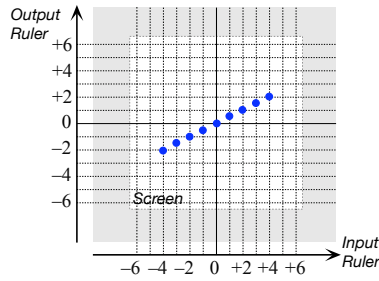
8.5. POWER ADD-ON FUNCTIONS

The resulting points are the plot point of *OUTPUT*

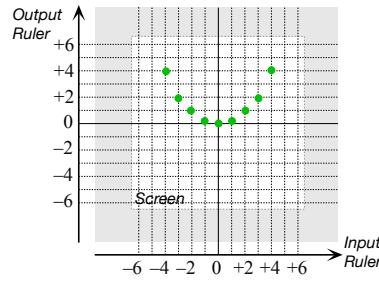


Altogether we have:

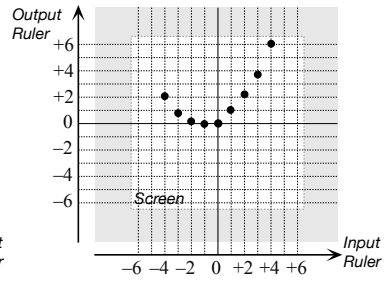
INPUT:



ADD-ON:



OUTPUT:



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