# MATH 161 CLASSWORK 9 Discussions


[ Run: 10/09/2014 at 23:35 Seed: 6477. Order of Checkable Items: List.]

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*Cw* **9-1.** Given the function $AMY$ specified by the global input-output rule

$$x \xrightarrow{AMY} AMY(x) = -3x + 6$$

find:

**i.** The *local rule* of $AMY$ near $\infty$
**ii.** The *local graph* of $AMY$ near $\infty$
**iii.** The *Height-sign* of $AMY$ near $\infty$
**iv.** The *Slope-sign* of $AMY$ near $\infty$

**Discussion:** An *affine* function is the result of *adding* (Chapter 8) two exceptional power functions. To do the local analysis of an affine function, the general idea is to "reduce" it to the local analysis of a *single* exceptional power function (Chapter 7) by taking advantage of the fact that the inputs are restricted to a *neighborhood*. Only the details of the local analysis will depend on where the center of the neighborhood is.

An extremely important aspect of the local analysis is the *language* that we use. *Which* details we write, and *how many*, depends on what we feel is important to help see the logical flow but we must constantly keep in mind is that NO loss of information should ever occur.

**i.** To get the *local* input-output rule of $AMY$ when $x$ is near $\infty$,

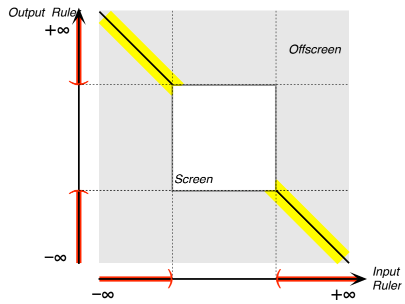
**a.** We *restrict* $x$ to being *large* (Observe that we are writing the *output* in order of *descending sizes*):

$$x\, large \xrightarrow{AMY} AMY(x) = -3x + 6$$

**b.** We keep only $-3x$ because, when the input is *large*, the size of $+6$ is too small for $+6$ to matter for the Height-sign and it doesn't affect the Slope-sign at all.

$$= -3x + [...]$$

**ii.** From the local input-output rule near $\infty$ of the exceptional power function $x \xrightarrow{DIL} DIL(x) = -3x$ we now get (Chapter 7) the local graph of $DIL$ near $\infty$:




wehere the black line is the local graph of the exceptional power function $DIL$ and where the yellow strip cntered on the black line corresponds to [...] in the local input-output rule and is thus the *place* of the local graph of $AMY$.

**iii.** From the local graph of the exceptional power function $DIL$ (seen from $\infty$), we see that Height-sign $AMY$ near $\infty = (-, +)$.

**iv.** From the local graph of the exceptional power function $DIL$ (seen from $\infty$), we see that Slope-sign $AMY$ near $\infty = (\diagdown, \diagdown)$.

*Cw* **9-2.** Given the function $SAM$ specified by the global input-output rule

$$x \xrightarrow{SAM} SAM(x) = +5x - 6$$

find:

**i.** The *local rule* of $SAM$ near 0
**ii.** The *local graph* of $SAM$ near 0
**iii.** The *Height-sign* of $SAM$ near 0
**iv.** The *Slope-sign* of $SAM$ near 0

**Discussion:** In its main lines, the local analysis near 0 proceeds exactly the same way as the local analysis near $\infty$ would.

**i.** To get the *local* input-output rule of $SAM$ when $x$ is near 0,

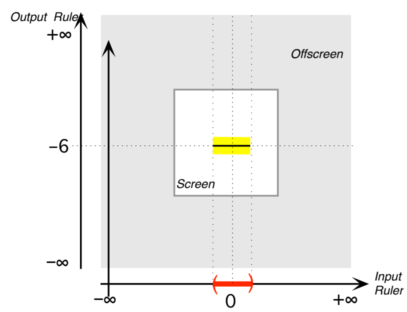
**a.** We *restrict* $x$ to being *small* which we indicate by using $h$ instead of $x$ (This is a universal code.) Observe that we are writing the *output* in order of *descending sizes*:

$$h \xrightarrow{SAM} SAM(h) = -6 + 5h$$

**b.** Here, we keep only $-6$ because, when the input is $h$, the size of $+5h$ is too small for $+5h$ to matter for the Height-sign (but, regardless of its small size, $+5h$ will matter for the Slope-sign so *then* we will have to put it back in).

$$= -6 + [...]$$

**ii.** From the local input-output rule near 0 of the exceptional power function $x \xrightarrow{CONST} CONST(x) = -6$ , we now get (Chapter 7) the local graph of $CONST$ near 0:



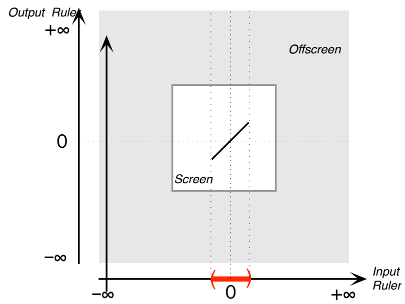

where the black line is the local graph of the exceptional power function $CONST$ and the yellow strip centered on the black line corresponds to [...] in the local input-output rule and is thus the *place* of the local graph of $SAM$.

**iii.** From the local graph of the exceptional power function $CONST$, we see that Height-sign $SAM$ near $0 = (-,-)$ (Regardless of exactly where the local graph of $SAM$ is in the yellow strip).

**iv.** From the local graph of the exceptional power function $CONST$), though, we cannot see Slope-sign $SAM$ near 0. To get Slope-sign $SAM$ near 0, we need the "full" local input-output rule

$$h \xrightarrow{SAM} SAM(h) = -6 + 5h$$

Actually, all we need to get Slope-sign $SAM$ near 0 is the graph of the exceptional power function $x \xrightarrow{DIL} DIL(x) = +5h$ near 0 (Chapter 7):




From that graph, we see that Slope-sign $SAM$ near $0 = (\diagup, \diagup)$

*Cw* **9-3.** Given the function $FRAN$ specified by the global input-output rule

$$x \xrightarrow{FRAN} FRAN(x) = +5x - 6$$

find:

**i.** The *local rule* of $FRAN$ near $-3$
**ii.** The *local graph* of $FRAN$ near $-3$
**iii.** The *Height-sign* of $FRAN$ near $-3$
**iv.** The *Slope-sign* of $FRAN$ near $-3$

**Discussion:** In its main lines, the local analysis near $-3$ proceeds exactly the same way as the local analysis near 0 would.

**i.** To get the *local* input-output rule of $FRAN$ when $x$ is near $-3$,

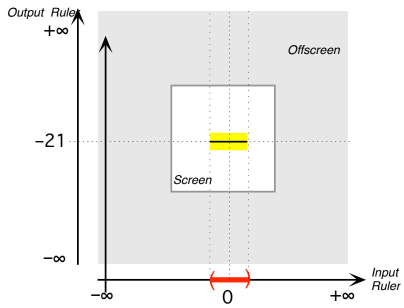
**a.** We *restrict* $x$ to being near $-3$ but we are now stuck since we cannot plug "$x$ near $-3$" in the output-specifying formula and compute. So, instead of using "$x$ near $-3$" we will use "$-3 + h$" where, remember, $h$ is *small*. The first thing to do then, will be to write the *output* in order of *descending sizes*:

$$\begin{aligned} -3 + h \xrightarrow{FRAN} FRAN(h) &= +5(-3 + h) - 6 \\ &= -15 + 5h - 6 \\ &= -21 + 5h \end{aligned}$$

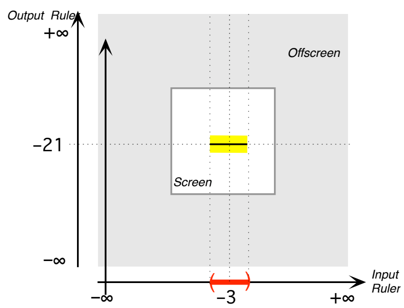
**b.** Here, we keep only $-21$ because, when the input is $-3 + h$, the size of $+5h$ is too small for $+5h$ to matter for the Height-sign (but, regardless of its small size, $+5h$ will matter for the Slope-sign so *then* we will have to put it back in).

$$= -21 + [...]$$

**ii.** From the local input-output rule near 0 of the exceptional power function $x \xrightarrow{CONST} CONST(x) = -21$, we get (Chapter 7) the local graph of $CONST$ near 0:




and therefore the local graph of $FRAN$ near $-3$



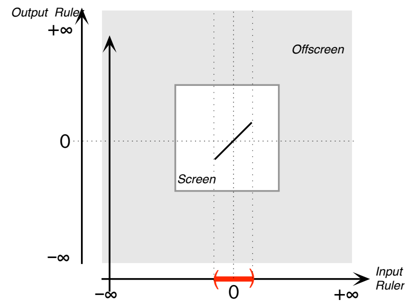

where the black line is the local graph of the exceptional power function $CONST$ and the yellow strip centered on the black line corresponds to [...] in the local input-output rule and thus is the *place* of the local graph of $FRAN$ near $-3$..

**iii.** From the local graph of the exceptional power function $CONST$, we see that Height-sign $FRAN$ near $-3 = (-,-)$ (Regardless of exactly where the local graph of $FRAN$ is in the yellow strip).

**iv.** From the local graph of the exceptional power function $CONST$), though, we cannot see Slope-sign $FRAN$ near $-3$. To get Slope-sign $FRAN$ near $-3$, we need the "full" local input-output rule

$$h \xrightarrow{FRAN} FRAN(h) = -21 + 5h$$

Actually, all we need to get Slope-sign $FRAN$ near $-3$ is the graph of the exceptional power function $x \xrightarrow{DIL} DIL(x) = +5h$ near 0 (Chapter 7):




From that graph, we see that Slope-sign $SAM$ near $0 = (\diagup, \diagup)$

*Cw* **9-4.** Given the function $MAC$ specified by the global input-output rule

$$x \xrightarrow{MAC} MAC(x) = -3x + 6$$

find:

**i.** The *local rule* of $MAC$ near $+2$

**ii.** The *local graph* of $MAC$ near $+2$

**iii.** The *Height-sign* of $MAC$ near $+2$

**iv.** The *Slope-sign* of $MAC$ near $+2$

**Discussion:** In its main lines, the local analysis near $+2$ proceeds exactly the same way as the local analysis near any bounded input would.

**i.** To get the *local* input-output rule of $MAC$ when $x$ is near $+2$,

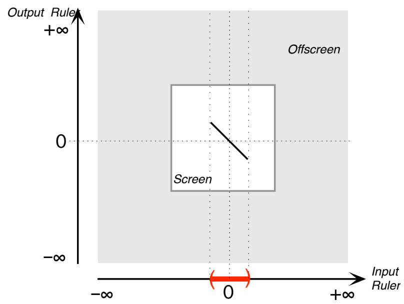
**a.** We *restrict* $x$ to being near $+2$ by using "$+2+h$" where, remember, $h$ is *small*. The first thing to do then, will be to write the *output* in order of *descending sizes*:

$$\begin{aligned} h \xrightarrow{MAC} MAC(h) &= -3(+2+h) + 6 \\ &= -6 - 3h + 6 \\ &= -3h \end{aligned}$$

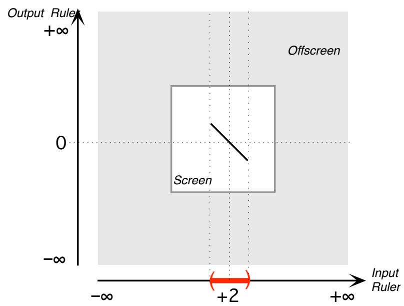
**b.** So, here, the constant power functions vanished by itself and we write

$$= -3h$$

**ii.** From the local input-output rule near 0 of the exceptional power function $h \xrightarrow{DIL} DIL(h) = -3h$, we get (Chapter 7) the local graph of $DIL$ near 0:




and therefore the local graph of $MAC$ near $+2$




where the black line is the local graph of the exceptional power function $DIL$ and therefore the local graph of $MAC$.

**iii.** From the local graph of $MAC$, we see that Height-sign $MAC$ near $+2 = (+, -)$ and that Slope-sign $MAC$ near $+2 = (\searrow, \searrow)$