Cw 9-1. Given the function $AMY$ specified by the global input-output rule
$$x \xrightarrow{AMY} AMY(x) = -3x + 6$$
find:

i. The local rule of $AMY$ near $\infty$
ii. The local graph of $AMY$ near $\infty$
iii. The Height-sign of $AMY$ near $\infty$
iv. The Slope-sign of $AMY$ near $\infty$

Discussion: An affine function is the result of adding (Chapter 8) two exceptional power functions. To do the local analysis of an affine function, the general idea is to “reduce” it to the local analysis of a single exceptional power function (Chapter 7) by taking advantage of the fact that the inputs are restricted to a neighborhood. Only the details of the local analysis will depend on where the center of the neighborhood is.

An extremely important aspect of the local analysis is the language that we use. Which details we write, and how many, depends on what we feel is important to help see the logical flow but we must constantly keep in mind is that NO loss of information should ever occur.

i. To get the local input-output rule of $AMY$ when $x$ is near $\infty$,

a. We restrict $x$ to being large (Observe that we are writing the output in order of descending sizes):
$$x \text{ large} \xrightarrow{AMY} AMY(x) = -3x + 6$$

b. We keep only $-3x$ because, when the input is large, the size of $+6$ is too small for $+6$ to matter for the Height-sign and it doesn’t affect the Slope-sign at all.
$$= -3x + [...]$$

ii. From the local input-output rule near $\infty$ of the exceptional power function $x \xrightarrow{DIL} DIL(x) = -3x$ we now get (Chapter 7) the local graph of $DIL$ near $\infty$: 
where the black line is the local graph of the exceptional power function
$DIL$ and where the yellow strip centered on the black line corresponds to
in the local input-output rule and is thus the place of the local graph
of $AMY$.

iii. From the local graph of the exceptional power function $DIL$ (seen
from $\infty$), we see that Height-sign $AMY$ near $\infty = (-, +)$.

iv. From the local graph of the exceptional power function $DIL$ (seen
from $\infty$), we see that Slope-sign $AMY$ near $\infty = (\backslash, \backslash)$.

9-2. Given the function $SAM$ specified by the global input-output rule

\[
\begin{align*}
x & \xrightarrow{SAM} \ SAM(x) = +5x - 6 
\end{align*}
\]

find:

i. The local rule of $SAM$ near 0
ii. The local graph of $SAM$ near 0
iii. The Height-sign of $SAM$ near 0
iv. The Slope-sign of $SAM$ near 0

Discussion: In its main lines, the local analysis near 0 proceeds exactly
the same way as the local analysis near $\infty$ would.

i. To get the local input-output rule of $SAM$ when $x$ is near 0,

\[a. \] We restrict $x$ to being small which we indicate by using $h$ instead
of $x$ (This is a universal code.) Observe that we are writing the
output in order of descending sizes:

\[
h \xrightarrow{SAM} \ SAM(h) = -6 + 5h
\]
b. Here, we keep only $-6$ because, when the input is $h$, the size of $+5h$ is too small for $+5h$ to matter for the Height-sign (but, regardless of its small size, $+5h$ will matter for the Slope-sign so then we will have to put it back in).

$$= -6 + \ldots$$

ii. From the local input-output rule near 0 of the exceptional power function $x \xrightarrow{\text{CONST}} \text{CONST}(x) = -6$, we now get (Chapter 7) the local graph of $\text{CONST}$ near 0:

![Graph of CONST near 0]

where the black line is the local graph of the exceptional power function $\text{CONST}$ and the yellow strip centered on the black line corresponds to $\ldots$ in the local input-output rule and is thus the place of the local graph of $\text{SAM}$.

iii. From the local graph of the exceptional power function $\text{CONST}$, we see that Height-sign $\text{SAM}$ near 0 = $(-, -)$ (Regardless of exactly where the local graph of $\text{SAM}$ is in the yellow strip).

iv. From the local graph of the exceptional power function $\text{CONST}$, though, we cannot see Slope-sign $\text{SAM}$ near 0. To get Slope-sign $\text{SAM}$ near 0, we need the “full” local input-output rule

$$h \xrightarrow{\text{SAM}} \text{SAM}(h) = -6 + 5h$$

Actually, all we need to get Slope-sign $\text{SAM}$ near 0 is the graph of the exceptional power function $x \xrightarrow{\text{DIL}} \text{DIL}(x) = +5h$ near 0 (Chapter 7):
From that graph, we see that Slope-sign $SAM$ near $0 = (\sqrt{\cdot}, /)$

**Ow 9-3.** Given the function $FRAN$ specified by the global input-output rule

$$x \xrightarrow{FRAN} FRAN(x) = +5x - 6$$

find:

i. The local rule of $FRAN$ near $-3$

ii. The local graph of $FRAN$ near $-3$

iii. The Height-sign of $FRAN$ near $-3$

iv. The Slope-sign of $FRAN$ near $-3$

**Discussion:** In its main lines, the local analysis near $-3$ proceeds exactly the same way as the local analysis near $0$ would.

i. To get the local input-output rule of $FRAN$ when $x$ is near $-3$,

<table>
<thead>
<tr>
<th>a. We restrict $x$ to being near $-3$ but we are now stuck since we cannot plug “$x$ near $-3$” in the output-specifying formula and compute. So, instead of using “$x$ near $-3$” we will use “$-3 + h$” where, remember, $h$ is small. The first thing to do then, will be to write the output in order of descending sizes:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3 + h \xrightarrow{FRAN} FRAN(h) = +5(-3 + h) - 6$</td>
</tr>
<tr>
<td>$= -15 + 5h - 6$</td>
</tr>
<tr>
<td>$= -21 + 5h$</td>
</tr>
</tbody>
</table>

b. Here, we keep only $-21$ because, when the input is $-3 + h$, the size of $+5h$ is too small for $+5h$ to matter for the Height-sign (but, regardless of its small size, $+5h$ will matter for the Slope-sign so then we will have to put it back in).

$$= -21 + \lfloor 5h \rfloor$$
ii. From the local input-output rule near 0 of the exceptional power function $x \xrightarrow{\text{CONST}} \text{CONST}(x) = -21$, we get (Chapter 7) the local graph of $\text{CONST}$ near 0:

![Local graph of CONST](image)

and therefore the local graph of $\text{FRAN}$ near $-3$

![Local graph of FRAN](image)

where the black line is the local graph of the exceptional power function $\text{CONST}$ and the yellow strip centered on the black line corresponds to $\ldots$ in the local input-output rule and thus is the place of the local graph of $\text{FRAN}$ near $-3$.

iii. From the local graph of the exceptional power function $\text{CONST}$, we see that Height-sign $\text{FRAN}$ near $-3 = (-, -)$ (Regardless of exactly where the local graph of $\text{FRAN}$ is in the yellow strip).

iv. From the local graph of the exceptional power function $\text{CONST}$, though, we cannot see Slope-sign $\text{FRAN}$ near $-3$. To get Slope-sign $\text{FRAN}$ near $-3$, we need the “full” local input-output rule

$$h \xrightarrow{\text{FRAN}} \text{FRAN}(h) = -21 + 5h$$

Actually, all we need to get Slope-sign $\text{FRAN}$ near $-3$ is the graph of the exceptional power function $x \xrightarrow{\text{DIL}} \text{DIL}(x) = +5h$ near 0 (Chapter 7):
From that graph, we see that Slope-sign $SAM$ near $0 = (/, /)$

**owe 9-4.** Given the function $MAC$ specified by the global input-output rule

$$x \xrightarrow{MAC} MAC(x) = -3x + 6$$

find:

i. The local rule of $MAC$ near $+2$

ii. The local graph of $MAC$ near $+2$

iii. The Height-sign of $MAC$ near $+2$

iv. The Slope-sign of $MAC$ near $+2$

**Discussion:** In its main lines, the local analysis near $+2$ proceeds exactly the same way as the local analysis near any bounded input would.

i. To get the local input-output rule of $MAC$ when $x$ is near $+2$,

    a. We restrict $x$ to being near $+2$ by using “$+2 + h$” where, remember, $h$ is small. The first thing to do then, will be to write the output in order of descending sizes:

    $$h \xrightarrow{MAC} MAC(h) = -3(+2 + h) + 6$$
    $$= -6 - 3h + 6$$
    $$= -3h$$

    b. So, here, the constant power functions vanished by itself and we write

    $$= -3h$$

ii. From the local input-output rule near $0$ of the exceptional power function $h \xrightarrow{DIL} DIL(h) = -3h$, we get (Chapter 7) the local graph of $DIL$ near $0$:
and therefore the local graph of $MAC$ near $+2$

where the black line is the local graph of the exceptional power function $DIL$ and therefore the local graph of $MAC$.

iii. From the local graph of $MAC$, we see that Height-sign $MAC$ near $+2 = (+,-)$ and that Slope-sign $MAC$ near $+2 = (\backslash,\backslash)$