MATH 161 QUIZ 12 Discussions

[Run: 03/16/2016 at 15:41 Seed: 6477. Order of Checkable Items: List.]

Qz **12-1.** Let f be the function specified by the global input-output rule

$$x \xrightarrow{\ f\ } f(x) = +2x^2 + 2x - 24$$

for which input(s), if any, is the output of f equal to 0?

Discussion:

APPROACH FROM SCRATCH. Since the global input-output rule which specifies f has three terms, we cannot "separate the variable from the constant" as is and we must first get rid of the middle term.

i. To do that we need to locate $x_{0-slope}$ and to do that we localize near x_0 to be specified later and we *declare* that x is to be replaced by $x_0 + u$

$$x_0 + u \xrightarrow{\ f\ } f(x_0+u) = +2(x_0+u)^2 + 2(x_0+u) - 24$$
$$= \Big[2x_0^2 + 2x_0 - 24\Big] + \Big[2 \cdot 2x_0 + 2\Big]u + \Big[+2\Big]u^2$$

Setting the coefficient of u equal to 0 gives $x_{0-slope} = -\frac{1}{2}$.

ii. We localize at $-\frac{1}{2}$ to get the global input-output rule using $\frac{-1}{2}$ as origin:

$$\tfrac{-1}{2} + u \xrightarrow{\ f\ } f(\tfrac{-1}{2}+u) = \Big[2\cdot(\tfrac{-1}{2})^2 + 2\cdot\tfrac{-1}{2} - 24\Big] + \Big[2\cdot 2\tfrac{-1}{2} + 2\Big]u\Big[+2\Big]u^2$$
$$= \Big[-24.5\Big] + \Big[0\Big]u + \Big[2\Big]u^2$$

iii. To locate $u_{\text{0-height}}$, we set $f(\frac{-1}{2}+u) = 0$, that is:

$$-24.5 + 2u^2 = 0$$
$$u^2 = 12.25$$
$$= \tfrac{49}{4}$$
$$u_{\text{0-height}} = \pm\tfrac{7}{2}$$

iv. Returning to global coordinates, since we declared that $x = x_0 + u$, we have that:

$$x_{\text{0-height}} = -\tfrac{1}{2} \pm \tfrac{7}{2}$$
$$= +3 \text{ or } -4$$

HALF MEMORY-BASED APPROACH. Having memorized that, for *quadratic* functions, $x_{0-slope} = \frac{-b}{2a}$ lets us start with step **ii.**

Full Memory-based Approach. Having memorized, in addition to $x_{0-slope} = \frac{-b}{2a}$, the **0-height Theorem 51** on page 206, after having computed that Discriminant=+4(49), we get

$$\begin{aligned} x_{\text{0-height}} &= -\tfrac{1}{2} \pm \tfrac{7}{2} \\ &= +3 \text{ or } -4 \end{aligned}$$

Qz **12-2.** Let the function JIM be specified by the global input-output rule

$$x \xrightarrow{JIM} JIM(x) = +6x^2 - 24$$

for which input(s), if any, is the output of JIM *negative*?

Discussion: We must get the solution subset of the *inequation*

$$JIM(x) < 0$$

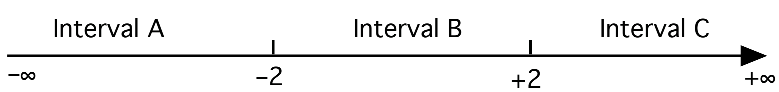
that is

$$+6x^2 - 24 < 0$$

We proceed with the usual three steps:

a. To get the boundary of the solution subset of the *inequation* $+6x^2-24 < 0$ we must get the solution subset of the *equation* $+6x^2-24 = 0$. Since the global input-output rule which specifies JIM has only *two* terms, we can "separate the variable from the constant" as is:

$$\begin{aligned} +6x^2 - 24 &= 0 \\ +6x^2 &= +24 \\ x^2 &= +4 \\ x &= \pm 2 \end{aligned}$$



b. To get the interior of the solution subset we test each one of the intervals:

- To test Interval A we pick an input far from the boundary so as to be able to use approximate computations. Using −1000, we get:

$$\begin{aligned} JIM(-1000) &< 0 \\ +6(-1000)^2 + [...] &< 0 \\ +6\,000\,000 + [...] &< 0 \end{aligned}$$

  which is certainly False.
- To test Interval B we cannot pick an input far enough from the boundary to use approximate computations. Here though, we are lucky since the boundary inputs have opposite signs so that we can

test interval B with 0 which simplifies the computations too.

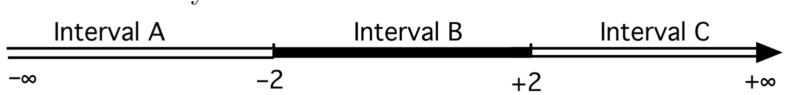
$$\begin{aligned} JIM(0) &< 0 \\ +6(0)^2 - 24 &< 0 \\ 0 - 24 &< 0 \\ -24 &< 0 \end{aligned}$$

which is certainly TRUE.

- To test Interval C we pick an input far from the BOUNDARY so as to be able to use approximate computations. Using +1000, we get:

$$\begin{aligned} JIM(+1000) &< 0 \\ +6(+1000)^2 + [...] &< 0 \\ +6\,000\,000 + [...] &< 0 \end{aligned}$$

which is certainly FALSE.



c. We finally test the BOUNDARY inputs themselves:

- We test −2:

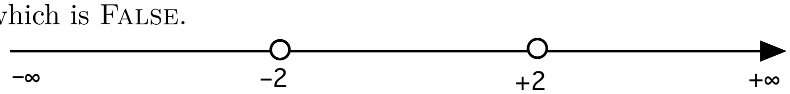
$$\begin{aligned} JIM(-2) &< 0 \\ +6(-2)^2 - 24 &< 0 \\ +6 \cdot +4 \ - 24 &< 0 \\ +24 - 24 &< 0 \\ 0 &< 0 \end{aligned}$$

which is FALSE.

- We test +2:

$$\begin{aligned} JIM(-2) &< 0 \\ +6(+2)^2 - 24 &< 0 \\ +6 \cdot +4 \ - 24 &< 0 \\ +24 - 24 &< 0 \\ 0 &< 0 \end{aligned}$$

which is FALSE.



Altogether then, using the Agreement that *non-solutions* are *not marked*, we have

Qz **12-3.** Let the function f be specified by the global input-output rule

$$x \xrightarrow{\ f\ } f(x) = -3x^2 + 12x + 5$$

for which input(s), if any, is Slope-sign $f = (\diagup, \diagup)$?

Discussion: Since the question is essentially the global version of the same question about the slope sign near, for instance, -5, we proceed in exactly the same manner with x_0 instead of -5, that is we localize near x_0 by declaring that x is to be replaced by $x_0 + h$ and focussing on the coefficient of h

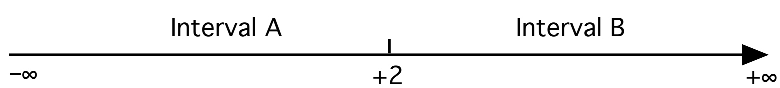
$$\begin{aligned} x_0 + h \xrightarrow{\quad f \quad} f(x_0+h) &= -3(x_0+h)^2 + 12(x_0+h) + 5 \\ &= \Big[\qquad\qquad \Big] + \Big[-3 \cdot 2x_0 + 12 \Big] h + \Big[\qquad \Big] h^2 \\ &= \Big[\qquad\qquad \Big] + \Big[-6x_0 + 12 \Big] h + \Big[\qquad \Big] h^2 \end{aligned}$$

For Slope-sign f near x_0 to be $(\diagup, \diagup)$, the coefficient of h must be *positive*. So we must get the solution subset of the *inequation* $-6x_0 + 12 > 0$

i. We get the BOUNDARY by getting the solution subset of the *equation* $-6x_0 + 12 = 0$ which is an affine equation so that we can solve by “separating the variable”:

$$\begin{aligned} -6x_0 + 12 &= 0 \\ -6x_0 &= -12 \\ x &= +2 \end{aligned}$$

So, we have $x_{0\text{-slope}} = +2$ and the BOUNDARY of the solution subset is:



ii. We get the INTERIOR of the solution subset by testing each one of the intervals:

- To test Interval A we pick an input far from the BOUNDARY so as to be able to use approximate computations. Using -1000, we get:

  $$\begin{aligned} -6(-1000) &> 0 \\ +6\,000\,000 &> 0 \end{aligned}$$

  which is TRUE.
- To test Interval B we pick an input far from the BOUNDARY so as to be able to use approximate computations. Using $+1000$, we get:

  $$\begin{aligned} -6(+1000) &> 0 \\ -6\,000\,000 &> 0 \end{aligned}$$

  which is FALSE.

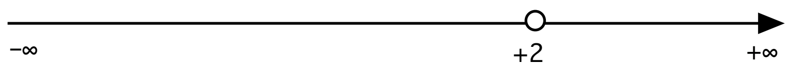
So, the INTERIOR of the solution subset is:

iii. We test the BOUNDARY input itself: We test +2:

$$-6 \cdot +2 > 0$$

$$-12 > 0$$

which is FALSE.



Altogether then, using the Agreement that *non-solutions* are *not marked*, the solution subset is: