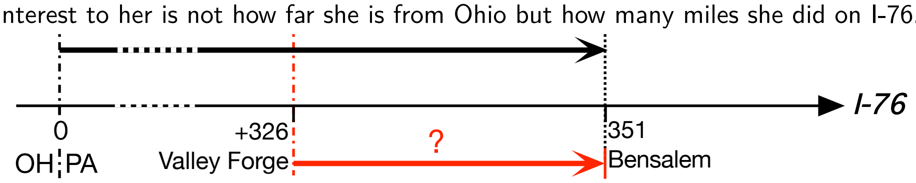
relative
localize
relative location

Chapter 21

Localization

Inputs are counted from the origin that comes with the ruler. However, rather than counting inputs **relative** to the origin of the ruler, it is often desirable to use some other origin to count inputs from.

EXAMPLE 21.1. Positions on I-76 in Pennsylvania are indicated by mileposts with milepost 0 at the Ohio border. Jane got onto I-76 at Valley Forge, where the milepost is 326. As she gets off I-76 at Bensalem she sees a milepost that says 351. But what is of interest to her is not how far she is from Ohio but how many miles she did on I-76.



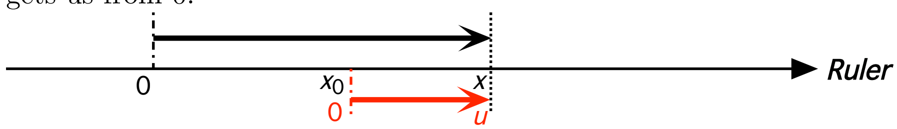
==========OK SO FAR==========

The origin 0 of the input ruler may not necessarily be very significant relative to a given function. Starting with affine functions, we will encounter many situations in which some number other than 0 plays a more important role than 0.

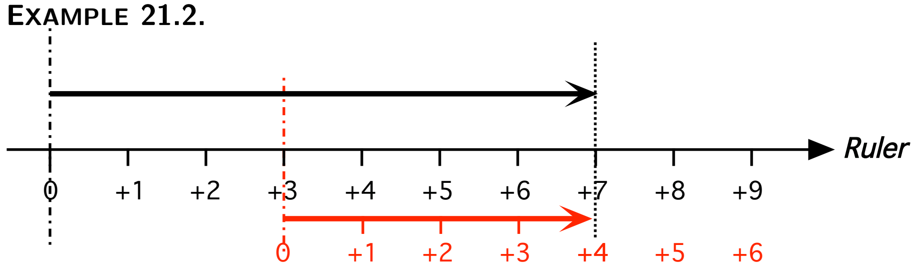
1. More precisely, given a function f and given an input x_0, we will often want to **localize** the function at x_0, that is count inputs from x_0 instead of from the origin 0 of the input ruler. In other words, we will use the **relative location** of x relative to x_0, that is we will replace in the global input-output rule x by $x_0 + u$ where u is the relative location of the input x.

location
local coordinate
global coordinate

location relative to x_0, also referred to as **local coordinate** (relative to x_0), is the number u that, from x_0, will get us to the same tickmark as x gets us from 0:



Example 21.2.



2.
3.

Example 21.3. Let f be the function specified by the global input-output rule

$$x \xrightarrow{f} f(x) = -3x + 12$$

If we locate inputs relative to $x_{\text{0-height}}$, that is relative to $+4$, that is if, instead of x we use $u = x - (+4)$, then the function f will be specified by the global input-output rule

$$u \xrightarrow{f} f(u) = -3u$$

In other words, while the function f is *affine* when the global input-output rule is specified relative to the origin 0 of the input ruler, the function f is *linear* when the global input-output rule is specified relative to $+4$ which is significant for f inasmuch as it is $x_{\text{0-height}}$. Note that $u_{\text{0-height}}$ is 0

The **global coordinate** of a tickmark is which side of the origin and how far from the origin 0 the tickmark it is. However, the global coordinate of a tickmark on a ruler may not be what we are interested in or need. Very often, we will want to use a local coordinate, that is the number

Given a function f, inputs for f are counted from the 0 on the ruler which however has no reason to be anywhere in particular in relation to the *global graph* of the function. More precisely, when the function f is specified by a global input-output rule,

0 is the origin of the ruler and thus all inputs are"counted" from 0. But inputs are "into a function" and so what matters most may not be how far from 0 an input is

This, though, may have something to do with the fact that inputs are counted from the 0 on the ruler which can be anywhere in relation to the *global graph* of the function, rather than from an input which is meaningful for the global graph of that function. locate

What we will do then is to try to use, instead of the inputs themselves, the *location* of the inputs relative to an input that is meaningful for the function at hand and the obvious thing is to try is $x_{0\text{-slope}}$ and so we will try to use:

=========OK SO FAR=========

EXAMPLE 21.4. When we input 0 into any regular power functions, the output is always 0 and thus the input 0 is likely to play an important role among the inputs of regular power functions.

But when we input 0 into the affine function specified by the global input-output rule,

a constant functions, the output is always the constant coefficient which is thus more likely to be the number to play a very special role.

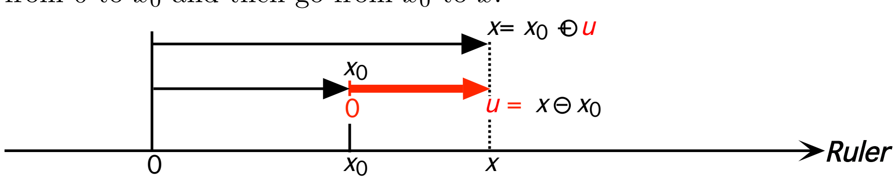
1. Given a quantitative ruler and

- given any bounded number, we can always *mark* it on the ruler (but usually only approximately),
- given any mark on the ruler, we can always find the bounded number whose mark it is, (but usually only approximately).

The quantitative ruler, though, gives an unwarranted importance to the number 0 since that is the number from which all other numbers are to be "counted" and we will then often want to **locate** other numbers x relative to some other number that plays a particular role in the situation.

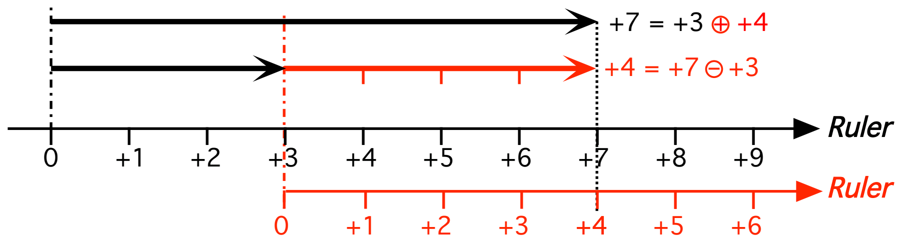
EXAMPLE 21.5. But, if she knows that Philadelphia is 434 miles from the Ohio border, then the mile marker is telling her that she is 34 miles away from home.

2. We now want to find the quantitative relationship between u and x which of course will depend on x_0: in order to go from 0 to x, we can go from 0 to x_0 and then go from x_0 to x.



In other words, first we go x_0 and then we go u.

EXAMPLE 21.6.


+7 = +3 ⊕ +4
+4 = +7 ⊖ +3
Ruler
0
+1
+2
+3
+4
+5
+6
+7
+8
+9
Ruler
0
+1
+2
+3
+4
+5
+6

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