equation problem
data set
equation
solution subset
solution
true
false
non-solution
REDUCTION METHOD
reduce
equation, original
equation, reduced
equivalent

Chapter 22

Reverse Problems

In order to deal with location problems, it is necessary to have a tool that works no matter what. The purpose here, then, is:

- To make sure of the algebra that we will need to deal with location problems,
- To have a systematic procedure to refer to,
- To set a language that is consistent

22.1 Equation Problems

1. We begin with the terminology that we will use to be extremely clear as to what we are doing. An **equation problem** will consist of:

i. A **data set** from which the number are to be picked/

EXAMPLE 22.1. If the problem to be dealt with involves the age of human beings, the data set should probably not involve negative numbers and probably not go much farther than about 100.

ii. An **equation** that will *specify* the **solution subset** of the *data set*, that is which will select the numbers in the data set that are to be retained as **solutions**, that is the numbers that turn the *equation* into a **true** sentence. Those numbers in the *data set* that turn the equation into a **false** sentence will be called **non-solutions**.

2. The approach that we will follow, which we will call the **REDUCTION METHOD**, will be to **reduce** the **original equation** to an equation until we get to an equation that we already know how to solve and we will call that equation the **reduced equation**. Of course, the *reduced equation* will have to be **equivalent** to the *original equation* in the sense that the *reduced equation* will have to have the same *solution subset* as the *original equation*.

affine equations

What we will do here will only be to add the same number to both sides or multiply both sides by the same number (other than 0) so that the following will apply:

THEOREM 22.1 Fairness . Given any *equation*, as long as, whatever we do onto one side of the verb =, we do exactly the same onto the other side of the verb =, we get an *equivalent* equation.

NOTE. While the **Fairness Theorem** seems obviously true, making the case that it is true is not that easy because what is not obvious is on what evidence to base the case. We will thus leave this issue for when the reader takes a course in MATHEMATICAL LOGIC.

3. In the case of **affine equations**, that is of equations of the form

$$ax + b = c$$

the REDUCTION METHOD proceeds as follows:

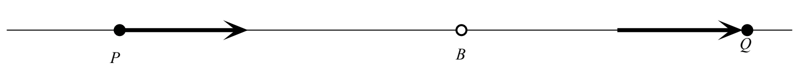
$$\begin{aligned} ax + b &= c \\ ax + b - b &= c - b \\ ax &= c - b \\ \frac{ax}{a} &= \frac{c-b}{a} \\ x &= \frac{c-b}{a} \end{aligned}$$

so that the **Fairness Theorem** applies and since $\frac{c-b}{a}$ is the solution of the reduced equation it is also the solution of the original equation.

22.2 Inequation Problems

This involves a general procedure that we will call the PASCH PROCEDURE (after the name of the mathematician who first noticed that, while it was quite obvious that in order to go from one side of a point on a straight line to the other side you had to "get across" the point, this turned out to be impossible to "prove" because the question was "on the basis of what" so that it had to be accepted as an axiom).

EXAMPLE 22.2. Given the point P on one side of the point B and the point Q on the other side of the point B, to go from P to Q, we need to get across the point B



1. Roughly, given an **inequation problem**, that is a **data set** and an **inequation**, in order to determine the **solution subset** of that *inequation problem*, we will proceed in two stages:

I. We will locate the **boundary** of the *solution subset* of the inequation problem by finding the *solution subset* of the **associated equation problem**.

(In the case of an *affine inequation*, we have already seen in the previous section how to deal with this stage.)

To graph the *boundary point* we will use:

- a **solid dot** when the boundary point is a *solution* of the *inequation*
- a **hollow dot** when the boundary point is a *non-solution* of the *inequation*.

II. We will locate the **interior** of the *solution subset* of the inequation problem, that is the solution subset of the associated **strict inequation problem**.

In the case of an *affine inequation*, the *boundary* consists of only one **boundary point** which separates the *data set* in two **sections** which we will call Section A and Section B. Then, we will locate the *interior* as follows:

i. We will pick a **test number** in Section A and check if the test number is a *solution* or a *non-solution* of the given inequation.

ii. We will pick a *test number* in Section B and check if the test number is a *solution* or a *non-solution* of the given inequation.

iii. We will then conclude with the help of

inequation problem
data set
inequation
solution subset
boundary
associated equation problem
solid dot
hollow dot
interior
strict inequation problem
boundary point
sections
test number

THEOREM 22.2 Pasch .

- If the *test number* in a section is a *solution*, then *all* numbers in that same section are *included* in the solution subset.
- If the *test number* in a section is a *non-solution*, then *all* numbers in that same section are *non-included* in the solution subset.

EXAMPLE 22.3. Given the inequation problem in which

- the *data set* consists of all numbers
- the *inequation* is

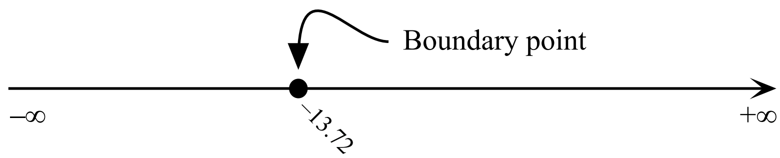
$$x \geqq -13.72$$

we locate separately.

i. The *boundary point* of the solution subset of the inequation problem is the solution of the *associated equation*:

$$x = -13.72$$

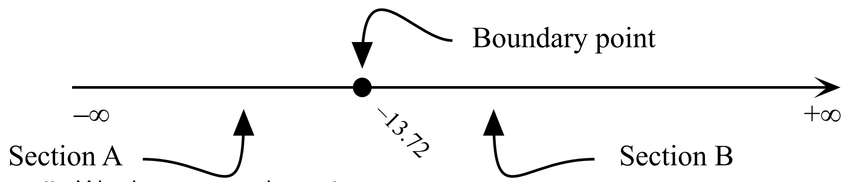
which, of course, is -13.72 and which we graph as follows since the boundary point is a *solution* of the inequation.



ii. The *interior* of the solution subset, that is the solution subset of the associated *strict inequation*

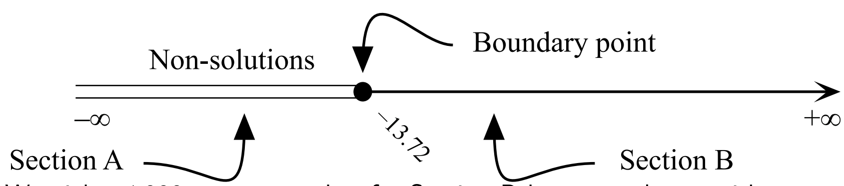
$$x > -13.72$$

i. The boundary point -13.72 separates the data set in two sections, Section A and Section B:

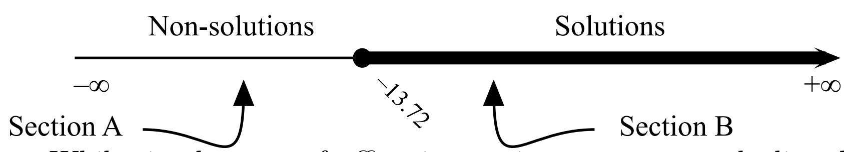


ii. We then test each section:

- We pick $-1\,000$ as test number for Section A because, almost without a glance we know $-1\,000$ is going to be in Section A and because it is easy to check in the inequation: we find that $-1\,000$ is a *non-solution* so that, by **Pasch Theorem**, all numbers in Section A are *non-solutions*.



- We pick $+1\,000$ as test number for Section B because, almost without a glance we know $+1\,000$ is going to be in Section B and because it is easy to check in the inequation: we find that $+1\,000$ is a *solution* so that, by **Pasch Theorem**, all numbers in Section A are *solutions*.



While, in the case of affine inequations, we can work directly on the inequation, this is not generally the case while, as we will see, the advantage of the Pasch Procedure is that it will work in all cases.

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