Chapter 25

Systems of Two First Degree Equations in Two Unknowns

25.1 General case

1. For an equation to be of degree 1 means that it involves only linear monomials.

$$\text{BOTH}\begin{cases} ax \oplus by = m \\ cx \oplus dy = n \end{cases}$$

Example 25.1.

$$\text{BOTH}\begin{cases} +3x \oplus +2y = +19 \\ -4x \oplus +5y = +13 \end{cases}$$

2. To eliminate one unknown, we multiply both sides of each equation by the coefficient of the unknown in the other equation and we *subtract* the resulting equations.

Example 25.2. To eliminate x in the following systems of equations,

$$\text{BOTH}\begin{cases} +3x \oplus +2y = +19 \\ -4x \oplus +5y = +13 \end{cases}$$

i. We multiply both sides of the first equation by -4 and both sides of the second equation by $+3$ in order for the coefficients of x to be the same:

$$-4\times\ [+3x \oplus +2y] = -4\times\ [+19]$$
$$+3\times\ [-4x \oplus +5y] = +3\times\ [+13]$$

We carry out the multiplications

$$-12x \oplus -8y = -76$$
$$-12x \oplus +15y = +39$$

ii. We *ominus* the second equation from the first equation to eliminate the x-terms:

$$\left[-12x \oplus -8y\right] = [-76]$$
$$\ominus \left[-12x \oplus +15y\right] \quad \ominus [+39]$$

that is we *oplus the opposite*

$$\left[-12x \oplus -8y\right] = [-76]$$
$$\oplus \left[+12x \oplus -15y\right] \quad \oplus [-39]$$

which gives us the equation

$$-23y = -115$$
$$\frac{-23}{-23}y = \frac{-115}{-23}$$
$$y = +5$$

3. To get the other unknown, we can either:

- Plug in $+5$ for y in either one of the original equations and solve for x. (This way is *shorter* but the disadvantage is that the computation of x depends on the result of the computation for y so an error for y will cause an error for x.)

or

- Eliminate y from the original equations just as we eliminated x. (This way is *longer* but the advantage is that the two computations are *independent* of each other so an error for y will not cause an error for x.)

25.2 Case for Affine Functions

When we are trying to determine the global input-output rule of an *affine* function A, we start from the fact that the global input-output rule is of the form

$$x \xrightarrow{A} A(x) = ax \oplus b$$

where the coefficients a and b are to be figured out.

1. When the affine function is specified as solution of an INITIAL VALUE PROBLEM, that is when we are given the *slope* and the *output* for one given input,

i. We replace a in the global input-output rule by the given *slope*,

ii. We declare that the input is the *given input*

$$\text{Given input} \xrightarrow{A} A(\text{given input}) = \text{given slope} \times \text{given input} \oplus b$$

and we write that the output is the *given output*

$$= \text{given output}$$

iii. So, we just need to solve for b the equation

$$\text{given slope} \times \text{given input} \oplus b = \text{given output}$$

EXAMPLE 25.3. Let the *affine* function JOE be the solution of the INITIAL VALUE PROBLEM

$$\text{BOTH}\begin{cases}\text{Slope at -2 is } +0.6\\ \text{Output at -2 is -30}\end{cases}$$

i. We replace a in the global input-output rule by $+0.6$:

ii. We declare that the input is-2

$$-2 \xrightarrow{A} JOE(-2) = +0.6 \times -2 \oplus b$$
$$= -1.2 \oplus b$$

and we write that the output is the given output

$$= -30$$

iii. So, we just need to solve for b the equation

$$-1.2 \oplus b = -30$$
$$b = +25$$

2. When the affine function is specified as solution of a BOUNDARY VALUE PROBLEM, that is when we are given the outputs for two different given inputs,

i. We declare that the input is the first given input and we write that the output is the corresponding given output.
ii. We declare that the input is the second given input and we write that the output is the corresponding given output.
iii. We solve the resulting system as in section 25.1 General case.

EXAMPLE 25.4. Let the *affine* function $JANE$ be the solution of the BOUNDARY VALUE PROBLEM

$$\text{BOTH}\begin{cases} \textit{JANE}\text{(-2)=+5} \\ \textit{JANE}\text{(+3)=-15} \end{cases}$$

i. Since the function $JANE$ is affine, the global input-output rule is of the form:

$$x \xrightarrow{JANE} JANE(x) = a \times x \oplus b$$

ii. We declare that the input is -2 and we write that the output is $+5$.

$$\begin{aligned} -2 \xrightarrow{JANE} JANE(-2) &= a \times -2 \oplus b \\ &= +5 \end{aligned}$$

so the first equation is

$$a \times -2 \oplus b = +5$$

iii. We declare that the input is $+3$ and we write that the output is -15.

$$\begin{aligned} -2 \xrightarrow{JANE} JANE(+3) &= a \times +3 \oplus b \\ &= -15 \end{aligned}$$

so the second equation is

$$a \times +3 \oplus b = -15$$

iv. We solve these two equations where the unknowns are a and b

$$\text{BOTH}\begin{cases} -2a \oplus b = +5 \\ +3x \oplus b = -15 \end{cases}$$

as in section 25.1 General case