Chapter 5

Regular Monomial Functions
- Local Analysis

Monomial functions are functions that multiply or divide a given number, referred to as the coefficient, by a number of copies of the input.

1. More precisely,

**DEFINITION 5.1. Monomial Functions** are algebraic functions whose global input-output rule is of the form

\[ f(x) = \text{coefficient} \cdot x^{\text{exponent}} \]

where:

- The **coefficient** can be any **bounded** number.
- The **exponent** in the **power** \(x^{\text{exponent}}\) is a **signed counting** number that specifies what the function is to do to the **coefficient** with the copies of \(x\):
  - The **size** of the exponent specifies **how many copies** of \(x\) are to be made. (If the exponent is 0, no copy is to be made and the coefficient is to be left alone.)
  - The **sign** of the exponent specifies whether the coefficient is to be multiplied or to be divided by the copies of \(x\).
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**5.1 Power Functions**

The name that is normally used for those monomial functions whose coefficient is \(+1\) or \(-1\). Unfortunately, the name power function is often used in place of monomial function and, even more unfortunately, this was the case in the previous editions of this text.

2. For reasons that will appear shortly we will distinguish:
   - The **regular monomial functions**, to be discussed in this and the next chapter, which are those monomial functions whose exponent is any signed counting number other than \(0\) or \(+1\).
   - The **exceptional monomial functions**, to be discussed in ??, which are those monomial functions whose exponent is either \(0\) or \(+1\).

**5.1 Output At** \(x_0\)

Let \(f\) be the regular monomial function specified by the global input-output rule

\[
\begin{align*}
\text{input} & \quad \overset{f}{\longrightarrow} \quad \text{output}\text{-specifying code} \\
\text{f} & = ax^{\pm n}
\end{align*}
\]

where \(n\) is the number of copies used by \(f\), and let \(x_0\) be the specified input. To get the output of the function \(f\) at the specified input \(x_0\), we use ?? on ?? which, for regular monomial functions, becomes:

**5.1 HOW TO Get the output at** \(x_0\) **of a regular monomial function** \(f\).

1. Declare that \(x\) is to be replaced by \(x_0\)

\[
\begin{align*}
\left. x \right|_{x \leftarrow x_0} & \overset{f}{\longrightarrow} \left. f(x) \right|_{x \leftarrow x_0} = ax^{\pm n} \left|_{x \leftarrow x_0} \\
\end{align*}
\]

which, once carried out, gives:

\[
\begin{align*}
\left. x_0 \right|_{x \leftarrow x_0} & \overset{f}{\longrightarrow} \left. f(x_0) \right|_{x \leftarrow x_0} = ax_0^{\pm n} \left|_{x \leftarrow x_0} \\
\end{align*}
\]

output-specifying code
5.1. Output At $x_0$

ii. **Execute** the output-specifying code that is:

a. **Decode** the output-specifying code, that is write out the computations to be performed according to the output-specifying code.

b. **Perform** the computations specified by the output-specifying code and thus get the output $f(x_0)$;

- For **positive** exponents, the code specifies that the output $f(x_0)$ is obtained by multiplying the coefficient $a$ by $n$ copies of the specified input $x_0$:
  \[
  f(x_0) = a \cdot x_0 \cdot \ldots \cdot x_0
  \]

- For **negative** exponents, the code specifies that the output $f(x_0)$ is obtained by dividing the coefficient $a$ by the $n$ copies of the specified input $x_0$:
  \[
  f(x_0) = \frac{a}{x_0 \cdot \ldots \cdot x_0}
  \]

**DEMO 5.1** Let $FLIP$ be the function specified by the global input-output rule

\[
    x \xrightarrow{FLIP} FLIP(x) = (+527.31)x^{+11}
\]

To get the output of the function $FLIP$ at $-3$:

i. We **declare** that $x$ is to be replaced by $-3$:

\[
    x \bigg|_{x=-3} \xrightarrow{FLIP} FLIP(x) \bigg|_{x=-3} = (+527.31)x^{+11} \bigg|_{x=-3}
\]

which, once the replacement has been carried out, gives:

\[
    -3 \xrightarrow{FLIP} FLIP(-3) = (+527.31) \cdot (-3)^{+11}
\]

ii. We **execute** the output-specifying code that is:

a. We **decode** the output-specifying code: since we have a **positive exponent**, the code specifies that the output $FLIP(-3)$ is obtained by multiplying the coefficient $+527.31$ by 11 copies of the specified input $-3$:

\[
    FLIP(-3) = (+527.31) \cdot (-3) \cdot \ldots \cdot (-3)
\]

11 copies of $-3$
b. We perform the computations specified by the code. Dealing separately with the \textit{signs} and the \textit{sizes}, we have
\[\left(527.31\right) \cdot \left(-\right) \cdot \ldots \cdot \left(-\right) \cdot \left(3\right) \cdot \ldots \cdot \left(3\right)\]
\[11 \text{ copies of } - \cdot \left(3\right) \cdot \ldots \cdot \left(3\right) \cdot \left(3\right) \cdot \ldots \cdot \left(3\right)\]
and since

- by ?? on ??, an \textit{odd} number of copies of $- \cdot$ multiply to $-$ and we get
\[\left(527.31\right) \cdot \left(-\right) \cdot \left(177 147\right)\]
\[= -93 411 384.57\]

The input-output pair is $(-3, -93 411 384.57)$

\textbf{Demo 5.2} Let $FLOP$ be the function specified by the global input-output rule
\[x \xrightarrow{FLOP} FLOP(x) = (+3.522.38)x^{-6}\]
To get the output of the function $FLOP$ at $-3$:

i. We declare that $x$ is to be replaced by $-3$
\[x \xrightarrow{x = -3} FLOP\]
\[\text{Output Specifying Code: } (+3.522.38) \cdot \left(-\right) \cdot \ldots \cdot \left(-\right) \cdot \left(3\right) \cdot \ldots \cdot \left(3\right)\]
\[6 \text{ copies of } - \cdot \left(3\right) \cdot \ldots \cdot \left(3\right) \cdot \left(3\right) \cdot \ldots \cdot \left(3\right)\]
which, once carried out, gives:
\[FLOP(-3) = (+3.522.38) \cdot \left(-\right) \cdot \ldots \cdot \left(-\right) \cdot \left(3\right) \cdot \ldots \cdot \left(3\right)\]

ii. We execute the output-specifying code that is:

- We decode the output-specifying code: since we have a \textit{negative exponent}, the code specifies that the output $FLOP(-3)$ is obtained by \textit{dividing} the coefficient $+3.522.38$ by $6$ copies of the specified input $-3$:
\[FLOP(-3) = \frac{+3.522.38}{\left(-\right) \cdot \ldots \cdot \left(-\right) \cdot \left(3\right) \cdot \ldots \cdot \left(3\right)\}
\[6 \text{ copies of } - \cdot \left(3\right) \cdot \ldots \cdot \left(3\right) \cdot \left(3\right) \cdot \ldots \cdot \left(3\right)\]

- We perform the computations specified by the code. Dealing separately with the \textit{signs} and the \textit{sizes}, we have
\[+3.522.38 \cdot \left(-\right) \cdot \ldots \cdot \left(-\right) \cdot \left(3\right) \cdot \ldots \cdot \left(3\right)\]
\[\text{even number of copies of } - \cdot \left(3\right) \cdot \ldots \cdot \left(3\right) \cdot \left(3\right) \cdot \ldots \cdot \left(3\right)\]
and since,

- by ?? on ??, an even number of copies of \( - \) multiply to \( + \) and we get

\[
\begin{align*}
+3522.38 \\
(+) \cdot (720) \\
= +4.8317 + [...]
\end{align*}
\]

The input-output pair is \((-3, +4.8317 + [...])\)

5.2 Plot Point

Let \( f \) be the regular monomial function specified by the global input-output rule

\[
\begin{align*}
\text{input} & \quad f \quad \text{output} \\
\longrightarrow & \quad f(x) = a x^{n} \\
\text{output-specifying code}
\end{align*}
\]

where \( n \) is the number of copies used by \( f \), and let \( x_0 \) be the specified input.
To plot the input-output pair for the specified input \( x_0 \), we use ?? on ??

which, in the case of regular monomial functions, becomes

**5.2 HOW TO Get the plot point for a specified bounded input**

1. Get the output at the specified input using How To 5.1 on page 78 to get the input-output pair,
2. Locate the plot point with How To 5.2 on page 81.

**DEMO 5.3** Let \( FLIP \) be the function specified by the global input-output rule

\[
\begin{align*}
x \quad \xrightarrow{FLIP} \quad FLIP(x) = (+527.31)x^{+11}
\end{align*}
\]

To plot the input-output pair for the input \(-3\):

1. We get the output of the function \( FLIP \) at \(-3\). We found in **EXAMPLE 5.1** above that \( FLIP(-3) = -93411.384.57 \)
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2. Thus, the input-output pair for the graph point of FLIP at $-3$ is $(-3, -93411384.57)$ and the plot point is:

3. Let $FLOP$ be the function specified by the global input-output rule

$$x \xrightarrow{FLOP} FLOP(x) = (+3522.38)x^{-6}$$

To plot the input-output pair for the input $-3$:

1. We get the output of the function $FLOP$ at $-3$. We found in Demo 5.2 on page 80 that $FLOP(-3) = +4.8317 + [...]$

2. Thus, the input-output pair for the graph point of $FLOP$ at $-3$ is $(-3, +4.8317 + [...])$ and the plot point is:

5.3 Normalization

Since in this text we will take a qualitative viewpoint, all the features of the global input-output rule that specifies a regular monomial function will not be equally important for us.

As we will see, the three features that will be important for us are:

- **Coefficient Sign** which can be $+$ or $-$.
- **Exponent Sign** which can be $+$ or $-$,
- **Exponent Parity** which can be even or odd depending on whether the size of the exponent, that is the number of copies, is even or odd.
5.3. Normalization

**Demo 5.5** The function specified by the global input-output rule

\[ x \xrightarrow{BLIP} BLIP(x) = (-160.42)x^7 \]

is a monomial function whose global input-output rule has the following features:
- Coefficient Sign \( BLIP = - \).
- Exponent Sign \( BLIP = + \).
- Exponent Parity \( BLIP = \text{odd} \).

But, because, in this text, we are only interested in qualitative analysis, we will not pay any attention to the following two features:
- **Coefficient Size** (other than the coefficient having to be **bounded**)
- **Exponent Size** (other than the size of the exponent being **even** or **odd**)

**NOTE.** A deeper analysis would require taking into account the actual number of copies but even then the size of the coefficient would still not matter much.

Accordingly, in order to focus on the important features of regular monomial functions, it will often be helpful to **normalize** the global input-output rule of a regular monomial function as follows:

**5.3 HOW TO Normalize the global I-O rule of a regular monomial function.**

i. Replace the **Coefficient Size** by the word **bounded**,  
ii. Replace the **Exponent Size** by the **Exponent Parity**

**Demo 5.6** Let \( BLIP \) be the function specified by the global input-output rule

\[ x \xrightarrow{BLIP} BLIP(x) = (-160.42)x^7 \]

To normalize \( BLIP \).

i. We replace the **Coefficient Size**, namely \( 160.42 \), by the word **bounded**  
ii. We replace the **Exponent Size**, namely \( 7 \), by the word **odd**

The normalized global input-output rule of \( BLIP \) is thus

\[ x \xrightarrow{BLIP} BLIP(x) = (-\text{bounded}) \cdot x^{\text{odd}} \]

**Demo 5.7** Let \( BLOP \) be the function specified by the global input-output rule

\[ x \xrightarrow{BLOP} BLOP(x) = (-365.28)x^{-6} \]

To normalize \( BLOP \).

i. We replace the **Coefficient Size**, namely \( 365.28 \), by the word **bounded**  
ii. We replace the **Exponent Size**, namely \( 6 \), by the word **even**
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The normalized global input-output rule of BLOP is thus

\[
x \xrightarrow{\text{BLOP}} \text{BLOP}(x) = (-\text{bounded}) \cdot x^{-\text{even}}
\]

5.4 Output Near $\infty$ and Near 0

As mentioned in ?? on ??, instead of using single inputs to get single plot points, we will “thicken the plot” that is we will use neighborhoods of given inputs to get graph places but to use neighborhoods with global input-output rules, we will first have to introduce code to be able to declare by what to replace $x$.

Since we are dealing here with regular monomial functions we will only be interested in inputs near $\infty$ and/or inputs near 0 and so here all we will need is the sign-size. But, since this at the very core of what we will be doing in the rest of this text, we want to proceed with the utmost caution.

1. When we want the output for inputs on either side of a neighborhood of $\infty$ or of 0:

   a. We use the following code to declare by what to replace $x$:

<table>
<thead>
<tr>
<th>Near</th>
<th>Side</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infinity</td>
<td>Left $0 \ldots \rightarrow \infty$ positive $+\infty$</td>
<td>$+\text{large}$</td>
</tr>
<tr>
<td></td>
<td>Right $\infty \ldots \rightarrow 0$ negative $-\infty$</td>
<td>$-\text{large}$</td>
</tr>
<tr>
<td>Zero</td>
<td>Left $\infty \ldots \rightarrow 0$ negative $0^-$</td>
<td>$-\text{small}$</td>
</tr>
<tr>
<td></td>
<td>Right $0 \ldots \rightarrow \infty$ positive $0^+$</td>
<td>$+\text{small}$</td>
</tr>
</tbody>
</table>

   b. Then, to get the input-output pairs we use How To 5.1 on page 78 but declare that $x$ is to be replaced using the above code instead of $x_0$:

   **5.4 HOW TO Get the input-output pairs near a side of $\infty$ or 0 for a regular monomial function.**

   1. Normalize the global input-input rule using How To 5.4 on page 84
   2. Declare that $x$ is to be replaced by $+\text{large}$ or $-\text{large}$ or $+\text{small}$
   3. Execute the output-specifying code that is:
      a. Decode the output-specifying code, that is write out the computations to be performed according to the output-specifying code.
      b. Perform the computations specified by the code using ?? on ?? and ?? on ?? or ?? on ??
Let $NADE$ be the function specified by the global input-output rule

$$x \xrightarrow{NADE} NADE(x) = (-83.91)x^{-5}$$

To get the input-output pairs near $+\infty$ for $NADE$:

i. We normalize $NADE$:

$$x \xrightarrow{NADE} NADE(x) = (-\text{bounded}) \cdot \text{odd}$$

ii. We declare that $x$ is to be replaced by $+\text{large}$

$$x \xrightarrow{x \rightarrow +\text{large}} NADE(x) = (-\text{bounded}) \cdot \text{odd} \xrightarrow{x \rightarrow +\text{large}}$$

which, once carried out, gives:

$$+\text{large} \xrightarrow{NADE} NADE(+\text{large}) = (-\text{bounded})(+\text{large}) \cdot \text{odd}$$

output-specifying code

iii. We execute the output-specifying code that is:

a. We decode the output-specifying code: since the exponent is negative, we get the output $NADE(+\text{large})$ by dividing the coefficient $-\text{bounded}$ by an odd number of copies of the specified input $+\text{large}$:

$$= -\text{bounded} \cdot \underbrace{(+\text{large}) \cdot \ldots \cdot (+\text{large})}_{\text{odd number of copies of } +\text{large}}$$

b. We perform the computations specified by the code. Dealing separately with the signs and the sizes, we have

$$= -\text{bounded} \cdot \underbrace{(+) \cdot \ldots \cdot (+)}_{\text{odd number of copies of } +} \cdot \underbrace{(\text{large}) \cdot \ldots \cdot (\text{large})}_{\text{odd number of copies of } \text{large}}$$

and since,

- by ?? on ??, any number of copies of $+$ multiply to $+$,
- by the Definition of $\text{large}$, any number of copies of $\text{large}$ multiply to $\text{large}$

$$= -\text{ bounded} \cdot + \cdot \text{large}$$

and by ?? on ?? and ?? on ?? we get

$$= -\text{ small}$$

iv. The input-output pairs are $(+\text{large}, -\text{small})$
Let \( MADE \) be the function specified by the global input-output rule

\[
x \xrightarrow{MADE} MADE(x) = (+27.61)x^{+5}
\]

To get the input-output pairs near \( 0^+ \) for \( MADE \):

i. We normalize \( MADE \):

\[
x \xrightarrow{MADE} MADE(x) = (+\text{bounded})x^{+\text{odd}}
\]

ii. We declare that \( x \) is to be replaced by \(+\text{small}\)

\[
x \xrightarrow{+\text{small}} MADE(x) \xrightarrow{+\text{small}} (+\text{bounded})x^{+\text{odd}} \xrightarrow{+\text{small}}
\]

which, once carried out, gives:

\[
+\text{small} \xrightarrow{MADE} MADE(+\text{small}) = (-\text{bounded})(+\text{small})^{\text{odd}}
\]

iii. We execute the output-specifying code that is:

a. We decode the output-specifying code: since the exponent is \( \text{positive} \), we get that the output \( MADE(+\text{small}) \) is obtained by \( \text{multiplying} \) the coefficient \( +\text{bounded} \) by an \( \text{odd} \) number of copies of the specified input \( +\text{small} \):

\[
= (+\text{bounded}) \cdot (+\text{small}) \cdot \ldots \cdot (+\text{small})
\]

b. We perform the computations specified by the code. Dealing separately with the \( \text{signs} \) and the \( \text{sizes} \), we have

\[
= (+\text{bounded}) \cdot (+\ldots\cdot (+) \cdot \text{odd number of copies of } +\cdot \text{odd number of copies of } +\text{small})
\]

and since,

- by the \( \text{Sign Multiplication Rule} \), any number of copies of \(+\) multiply to \(+\)
- by the \( \text{Definition} \) of \( \text{small} \), any number of copies of \( \text{small} \) multiply to \( \text{small} \)

\[
= (+\text{bounded}) \cdot +\text{small}
\]

and by the \( \text{Sign Multiplication Rule} \) and the \( \text{Size Multiplication Theorem} \)

\[
= +\text{small}
\]

iv. The input-output pairs are \((+\text{small}, -\text{small})\)
5.4. Output Near \( \infty \) and Near 0

**Demo 5.10** Let \( RADE \) be the function specified by the global input-output rule
\[
x \xrightarrow{RADE} RADE(x) = (45.67)x^{-4}
\]

To get the input-output pairs near \(-\infty\) for \( RADE \):

i. **We normalize** \( RADE \):
\[
x \xrightarrow{RADE} RADE(x) = (+\text{bounded})x^{\text{even}}
\]

ii. **We declare** that \( x \) is to be replaced by \(-\text{large}\)
\[
x \xrightarrow{x \rightarrow -\text{large}} RADE(x) \xrightarrow{x \rightarrow -\text{large}} (+\text{bounded})x^{\text{even}}
\]

which, once carried out, gives:
\[
-large \xrightarrow{RADE} RADE(-large) = (+\text{bounded})(-large)^{\text{even}}
\]

iii. **We execute** the output-specifying code that is:

a. **We decode** the output-specifying code: since the exponent is \( -\text{negative} \),
we get the output \( RADE(-\text{large}) \) by **dividing** the coefficient \( +\text{bounded} \) by an \( \text{even} \) number of copies of the specified input \(-\text{large}\):
\[
(+\text{bounded}) \cdot \ldots \cdot (-\text{large})^{\text{even number of copies of } -\text{large}}
\]

b. **We perform** the computations specified by the code:

Dealing separately with the **signs** and the **sizes**, we have
\[
(+\text{bounded}) \cdot \ldots \cdot (-) \cdot \ldots \cdot (-) \cdot \ldots \cdot (large) \cdot \ldots \cdot (large)
\]

and since,
- by the **Sign Multiplication Rule**, any \( \text{even} \) number of copies of \(-\) multiply to \(+\)
- by the **Definition** of \( \text{large} \), any number of copies of \( \text{large} \) multiply to \( \text{large} \)
\[
(+\text{bounded}) \cdot \ldots \cdot (-) \cdot \ldots \cdot (+) \cdot \ldots \cdot (large) \cdot \ldots \cdot (large) = +\text{small}
\]

and by the **Sign Division Rule** and the **Size Division Theorem**
\[
(+\text{small})
\]

iv. The input-output pairs are \((-\text{large}, +\text{small})\)
Let \( W A D E \) be the function specified by the global input-output rule
\[
x \xrightarrow{W A D E} W A D E(x) = (-28.34)x^{-3}
\]

To get the output of \( W A D E \) near 0+

i. We normalize \( W A D E \):
\[
x \xrightarrow{W A D E} W A D E(x) = (-\text{bounded})x^{-\text{even}}
\]

ii. We declare that \( x \) is to be replaced by + small
\[
x \xrightarrow{W A D E} W A D E(x) = (-\text{bounded})x^{-\text{even}} \quad \Big|_{x \leftarrow \text{+ small}}
\]

which, once carried out, gives:
\[
+\text{small} \xrightarrow{W A D E} W A D E(+\text{small}) = (-\text{bounded})(+\text{small})^{-\text{even}}
\]

iii. We execute the output-specifying code that is:

a. We decode the output-specifying code: since the exponent is negative, we get the output \( W A D E(+\text{small}) \) by dividing the coefficient \( -\text{bounded} \) by an even number of copies of the specified input + small:
\[
= -\text{bounded} \bigg/ \underbrace{(+\text{small}) \cdot \ldots \cdot (+\text{small})}_{\text{even number of copies of } +\text{small}}
\]

b. We perform the computations specified by the code. Dealing separately with the signs and the sizes, we have
\[
= -\text{bounded} \cdot \underbrace{(+ \cdot \ldots \cdot +)}_{\text{even number of copies of } +} \cdot \underbrace{\text{small} \cdot \ldots \cdot \text{small}}_{\text{even number of copies of } \text{small}}
\]

and since,
- by the Sign Multiplication Rule, any number of copies of + multiply to +
- by the Definition of small, any number of copies of small multiply to small
\[
= -\text{bounded} \cdot \underbrace{+ \cdot \text{small}}_{\text{large}}
\]

and by the Sign Division Rule and the Size Division Theorem
\[
= -\text{large}
\]

iv. The input-output pairs are (+ small, large)

2. When we want the output for inputs on both side of a neighborhood of \( \infty \) or of 0, all we need to do is to deal with each side as above. However
we cannot afford to do each side completely separately and, in order to deal
with both sides at the same time:

a. We use the bi-level signs ± and ± as follows:

i. We use ± to keep track of the inputs on each side of ∞ or 0,

ii. But, to keep track of the sign of the outputs on each side and to write
the input-output pairs,

• If the outputs on each side have different signs, we use ± or ± de-
pending on the result of the computations,

• If the outputs on each side have the same sign, + or −, we write just
that sign.

Example 5.1.

• If +small gives +large and −small gives −large,
then we write (±small, ±large) for the input-output pairs.

• If +small gives −large and −small gives +large,
then we write (±small, ±large) for the input-output pairs.

• If +small gives −large and −small gives −large too,
then we write (±small, −large) for the input-output pairs.

b. Then, we get the input-output pairs on each side of ∞ or 0 by
declaring that x is to be replaced by ±large or ±small and keeping
track of the signs as we perform the computations specified by the output-
specifying code.

Demo 5.12  Let PADE be the function specified by the global input-output
rule

\[
x \xrightarrow{\text{PADE}} \text{PADE}(x) = (-65.18)x + 6
\]

To get the input-output pairs near ∞ for PADE

i. We normalize PADE.

\[
x \xrightarrow{\text{PADE}} \text{PADE}(x) = (-\text{bounded}) x^{+\text{even}}
\]

ii. We declare that x is to be replaced by ±large

\[
x \bigg|_{x \leftarrow \pm\text{large}} \xrightarrow{\text{PADE}} \text{PADE}(x) \bigg|_{x \leftarrow \pm\text{large}} = (-\text{bounded})x^{+\text{even}} \bigg|_{x \leftarrow \pm\text{large}}
\]

which, once carried out, gives:

\[
\pm\text{large} \xrightarrow{\text{PADE}} \text{PADE}(\pm\text{large}) = (-\text{bounded})(\pm\text{large})^{+\text{even}}
\]

iii. We execute the output-specifying code that is:
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a. We decode the output-specifying code: since the exponent is positive, we get the output \( \text{PADE}(\pm \text{large}) \) by multiplying the coefficient \( \pm \text{bounded} \) by an even number of copies of the specified input \( \pm \text{large} \):

\[
= (-\text{bounded}) \cdot (\pm \text{large}) \cdot \ldots \cdot (\pm \text{large})
\]

b. We perform the computations specified by the code. Dealing separately with the signs and the sizes, we have

\[
= (-\text{bounded}) \cdot (\pm) \cdot \ldots \cdot (\pm) \cdot (\text{large}) \cdot \ldots \cdot (\text{large})
\]

and since,

- by the Sign Multiplication Rule, an even number of copies of + multiply to + and an even number of copies of − multiply to +
- by the Definition of large, any number of copies of large multiply to large

\[
= (-\text{bounded}) \cdot (+\cdot \text{large})
\]

and by the Sign Division Rule and the Size Division Theorem

\[
= -\text{large}
\]

iv. The input-output pairs are \((\pm \text{large}, -\text{large})\)

\[\text{Demo 5.13}\]

Let \( JADE \) be the function specified by the global input-output rule

\[
x \xrightarrow{JADE} JADE(x) = (-65.71)x^{-5}
\]

To get the output of \( JADE \) near 0,

i. We normalize \( JADE \):

\[
x \xrightarrow{JADE} JADE(x) = (-\text{bounded})x^{-\text{odd}}
\]

ii. We declare that \( x \) is to be replaced by \( \pm \text{small} \)

\[
x \xleftarrow{x \leftrightarrow \pm \text{small}} \xrightarrow{JADE} JADE(x) \xrightarrow{x \leftrightarrow \pm \text{small}} \left((-\text{bounded})x^{-\text{odd}}\right)_{x \leftrightarrow \pm \text{small}}
\]

which, once carried out, gives:

\[
\pm \text{small} \xrightarrow{JADE} JADE(\pm \text{small}) = (-\text{bounded})(\pm \text{small})^{\text{odd}}
\]

iii. We execute the output-specifying code that is:
a. We decode the output-specifying code: since the exponent is negative, we get the output \( JADE(\pm \text{small}) \) by dividing the coefficient \(-\text{bounded}\) by an odd number of copies of the specified input \( \pm \text{small} \):

\[
\frac{-\text{bounded}}{(\pm \text{small}) \cdots (\pm \text{small})} = \text{bounded}
\]

b. We perform the computations specified by the code. Dealing separately with the signs and the sizes, we have

\[
\frac{-\text{bounded}}{(\pm) \cdots (\pm)} \cdot \frac{(\text{small}) \cdots (\text{small})}{\text{odd number of copies of } \pm \text{small}} = \pm \text{large}
\]

and since,

- by the Sign Multiplication Rule, an odd number of copies of + multiply to + and an odd number of copies of − multiply to −
- by the Definition of small, any number of copies of small multiply to small

\[
\frac{-\text{bounded}}{\pm \text{small}} = \mp \text{large}
\]

and by the Sign Division Rule and the Size Division Theorem

iv. The input-output pairs are \( (\pm \text{small}, \mp \text{large}) \)

---

**DEMO 5.14** Let \( DADE \) be the function specified by the global input-output rule

\[
x \xrightarrow{DADE} DADE(x) = (-83.91)x + 5
\]

To get the input-output pairs near \( \infty \) for \( DADE \):

i. We normalize \( DADE \):

\[
x \xrightarrow{DADE} DADE(x) = (-\text{bounded})x^{+\text{odd}}
\]

ii. We declare that \( x \) is to be replaced by \( \pm \text{large} \)

\[
x \xrightarrow{\pm \text{large}} DADE(\pm \text{large}) = (-\text{bounded})(\pm \text{large})^{+\text{odd}}
\]

which, once carried out, gives:

\[
\pm \text{large} \xrightarrow{DADE} DADE(\pm \text{large}) = (-\text{bounded})(\pm \text{large})^{+\text{odd}}
\]

iii. We execute the output-specifying code that is:
a. We decode the output-specifying code: since the exponent is positive, we get that the output $DADE(\pm \text{large})$ is obtained by multiplying the coefficient $-\text{bounded}$ by an odd number of copies of the specified input $\pm \text{large}$:

$$= (-\text{bounded}) \cdot (\pm \text{large}) \cdot \ldots \cdot (\pm \text{large})$$

odd number of copies of $\pm \text{large}$

b. We perform the computations specified by the code. Dealing separately with the signs and the sizes, we have

$$= (-\text{bounded}) \cdot (\pm \cdot \ldots \cdot (\pm) \cdot (\text{large} \cdot \ldots \cdot (\text{large}$$

and since,

- by the Sign Multiplication Rule, an odd number of copies of $+$ multiply to $+$ and an odd number of copies of $-$ multiply to $-$
- by the Definition of $\text{large}$, any number of copies of $\text{large}$ multiply to $\text{large}$

$$= (-\text{bounded}) \cdot \pm \cdot \text{large}$$

and by the Sign Multiplication Rule and the Size Multiplication Theorem

$$= \mp \text{large}$$

iv. The input-output pairs are $(\pm \text{large}, \mp \text{large})$

---

**Demo 5.15** Let $FADE$ be the function specified by the global input-output rule

$$x \xrightarrow{FADE} FADE(x) = (-65.18)x^+6$$

To get the input-output pairs near $0$ for $FADE$:

i. We normalize $FADE$.

$$x \xrightarrow{FADE} FADE(x) = (-\text{bounded}) x^{+\text{even}}$$

ii. We declare that $x$ is to be replaced by $\pm \text{small}$

$$x \bigg|_{x = \pm \text{small}} \xrightarrow{FADE} FADE(x) \bigg|_{x = \pm \text{small}} = (-\text{bounded})x^{+\text{even}} \bigg|_{x = \pm \text{small}}$$

which, once carried out, gives:

$$\pm \text{small} \xrightarrow{FADE} FADE(\pm \text{small}) = (-\text{bounded})(\pm \text{small})^{+\text{even}}$$

output-specifying code

iii. We execute the output-specifying code that is:
5.5 Graph Place Near $\infty$ and Near 0

Once we have the input-output pairs near $\infty$ and near 0, we get the graph places as in ?? on ??.

i. In the first four demos, Demo 5.16, Demo 5.17, Demo 5.18, Demo 5.19, we will deal with only one side or the other.

ii. In the next four demos, Demo 5.20, Demo 5.21, Demo 5.22, Demo 5.23, we will deal with both sides at the same time.

5.5 How To Locate the graph place near $\infty$ or 0

1. Get the output of the function using How To 5.4 on page 84 to get the input-output pairs.

2. Locate the graph place using ?? on ??.
Chapter 5. Regular Monomial Functions - Local Analysis

**Demo 5.16** Let $NADE$ be the function specified by the global input-output rule

\[ x \xrightarrow{NADE} NADE(x) = (-83.91)x^{-5} \]

To locate the graph place of $NADE$ near $+\infty$:

1. We get that the input-output pairs for $NADE$ near $+\infty$ are (+large, −small) (See Demo 5.8 on page 85)

2. The graph place of $NADE$ near $+\infty$ then is:

![Diagram of graph place for NADE near +\infty](image)

**Demo 5.17** Let $MADE$ be the function specified by the global input-output rule

\[ x \xrightarrow{MADE} MADE(x) = (+27.61)x^{+5} \]

To locate the graph place of $MADE$ near $0^+$:

1. We get that the input-output pairs for $MADE$ near $0^+$ are [+small, +small] (See Demo 5.9 on page 86)

2. The graph place of $MADE$ near $0^+$ then is:

![Diagram of graph place for MADE near 0^+](image)
5.5. Graph Place Near $\infty$ and Near 0

**DEMO 5.18** Let $RADE$ be the function specified by the global input-output rule

$$x \xrightarrow{\text{RADE}} RADE(x) = (+45.67)x^{-4}$$

To locate the graph place of $RADE$ near $-\infty$:

1. We get that the input-output pairs for $RADE$ near $-\infty$ are $[-\text{large}, +\text{small}]$ (See Demo 5.10 on page 87)
2. The graph place near $-\infty$ then is:

**DEMO 5.19** Let $WADE$ be the function specified by the global input-output rule

$$x \xrightarrow{\text{WADE}} WADE(x) = (-28.34)x^{-3}$$

To locate the graph place of $WADE$ near $0^+$:

1. We get that the input-output pairs for $WADE$ near $0^+$ are $[+\text{small}, -\text{large}]$ (See Demo 5.11 on page 88)
2. The graph place near $0^+$ then is:
**Demo 5.20** Let \( PADE \) be the function specified by the global input-output rule
\[
x \xrightarrow{PADE} PADE(x) = (-65.18)x^6
\]

To locate the graph place of \( PADE \) near \( \infty \):

1. We get that the input-output pairs for \( PADE \) near \( \infty \) are \([\pm \text{large}, -\text{large}]\) (See Demo 5.12 on page 89)

2. The graph place of \( PADE \) near \( \infty \) then is:

**Demo 5.21** Let \( JADE \) be the function specified by the global input-output rule
\[
x \xrightarrow{JADE} JADE(x) = (-65.71)x^{-5}
\]

To locate the graph place of \( JADE \) near \( 0 \):

1. We get that the input-output pairs for \( JADE \) near \( 0 \) are \([\pm \text{small}, +\text{large}]\) (See Demo 5.13 on page 90)

2. The graph place of \( JADE \) near \( 0 \) then is:
5.6. Local Graph Near $\infty$ and Near 0

Regular monomial functions are very nice in that the shapes of the local graphs near $\infty$ and near 0 are forced by the graph place. In other words,
once we know the graph place, there is only one way we can draw the local graph because:

i. The smaller or the larger the input is, the smaller or the larger the output will be,

ii. The local graph cannot escape from the place.

**Demo 5.24** Given a monomial function for which the place of a local graph is \([+\text{large}, +\text{small}]\), we get the shape of the local graph as follows

i. The slope is forced by the fact that the larger the input is, the smaller the output will be.

ii. The concavity is forced by the fact that the local graph cannot cross the 0-output level line.

**Demo 5.25** Given a monomial function for which the place of a local graph is \([-\text{small}, -\text{large}]\), we get the shape of the local graph as follows

i. The slope is forced by the fact that the smaller the input is, the larger the output will be.

ii. The concavity is forced by the fact that the local graph cannot cross the 0-input level line.

### 5.7 Local Features Near \(\infty\) and Near 0

1. Given a regular monomial function being specified by a global input-output rule, to get the Height sign near \(\infty\) or near 0, we need only compute the sign of the outputs for nearby inputs with the global input-output rule.
5.7. Local Features Near $\infty$ and Near 0

**Demo 5.26** Let $JOE$ be the function specified by the global input-output rule

$$x \xrightarrow{JOE} JOE(x) = (-65.18)x^6$$

To get the Height sign of $JOE$ near $0^+$

We ignore the size and just look at the sign:

$$+ \xrightarrow{JOE} JOE(+) = (-)(+)^6$$

$$= (-) \cdot (+)$$

$$= -$$

and

$$- \xrightarrow{JOE} JOE(-) = (-)(-)^6$$

$$= (-) \cdot (+)$$

$$= -$$

So, Height sign $JOE$ near 0 is $(-, -)$

2. Given a regular monomial function being specified by a global input-output rule, to get the Slope sign or the Concavity sign near $\infty$ or near 0, we need the local graph near $\infty$ or near 0.

**Demo 5.27** Let $JILL$ be the function specified by the global input-output rule

$$x \xrightarrow{JILL} JILL(\pm) = (+32.06)(\pm)^6$$

To get the Slope sign of $JILL$ near 0

We need the local graph of $JILL$ near 0.

i. We get the output for $JILL$ near 0

$$\pm \text{small} \xrightarrow{JILL} JILL(\pm \text{small})$$

$$= (+\text{bounded})(\pm \text{small})^{\text{even}}$$

$$= (+\text{bounded})(\pm)^{\text{even}}(\text{small})^+$$

$$= (+\text{bounded})(+) \cdot (\text{small})$$

$$= +\text{small}$$

ii. The local graph of $JILL$ near 0 is

```
+∞ Outputs
-∞

-∞ -small +small Inputs
```

iii. Slope sign $JILL$ near 0 = $\langle \wedge, \vee \rangle$
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**Demo 5.28** Let \( JIM \) be the function specified by the global input-output rule

\[
JIM(x) = (-72.49)x^{-5}
\]

To get the **Concavity sign** of \( JIM \) near \( \infty \),

i. We get the output for \( JIM \) near \( \infty \).

\[
\pm\text{large} \xrightarrow{\text{JIM}} \text{JIM}(\pm\text{large})
\]

\[
= (-\text{bounded})(\pm\text{large})^{-\text{odd}}
\]

\[
= \frac{-\text{bounded}}{(\pm\text{large})\ldots(\pm\text{large})}
\quad \text{odd number of copies}
\]

\[
= -\text{bounded}
\]

\[
\pm\text{large}
\]

\[
= -\text{bounded} \cdot \pm\text{small}
\]

\[
= \mp\text{small}
\]

ii. The local graph of \( JIM \) near 0 is

iii. Concavity sign \( JIM \) near \( \infty = (\cap, \cup) \)
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