Chapter 8

Prelude To Polynomial Functions

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As already mentioned, monomial functions will be the building blocks from which all the functions we will be investigating in this text are built from. So we will always have to use more than a single monomial function at a time.

8.1 Adding Functions

1. Given a function, to which we will refer as base function, one often needs to add a number to each output that the base function returns. Whether or not this add-on number remains the same regardless of the input or differs depending on the input, we can look upon the add-on number as being itself the output returned for the same input by some other function to which we will refer as add-on function. (If the add-on number is the same regardless of the input this just means that the add-on function is a constant function.)

There is then going to be a third function, to be referred as sum function, which, for each input, will return the output returned by the base function plus the add-on number returned by the add-on function for that input.

In other words, given the two functions
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bar graph

\[ x \xrightarrow{\text{BASE}} \text{BASE}(x) \]

and

\[ x \xrightarrow{\text{ADD-ON}} \text{ADD-ON}(x) \]

there will be a third function specified as

\[ x \xrightarrow{\text{SUM}} \text{SUM}(x) = \text{BASE}(x) + \text{ADD-ON}(x) \]

2. In sciences such as Biology, Psychology and Economics the three functions are often in tabular form.

**Example 8.1.** When we shop online for, say for a textbook, we first see a price list—the base function. However, a shipping charge, which might or might not depend on the textbook, is usually added-on to the list price and is given by the shipping charge list—the add-on function. The price we end-up having to pay is thus given by the actual price list—the sum function.

<table>
<thead>
<tr>
<th>( x \xrightarrow{\text{LIST}} \text{LIST}(x) )</th>
<th>( x \xrightarrow{\text{SHIP}} \text{SHIP}(x) )</th>
<th>( x \xrightarrow{\text{PAY}} \text{PAY}(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>English 140</td>
<td>English 13.15</td>
<td>English 140 + 13.15 = 153.15</td>
</tr>
<tr>
<td>History 80</td>
<td>History 3.45</td>
<td>History 80 + 3.45 = 83.45</td>
</tr>
<tr>
<td>Biology 130</td>
<td>Biology 7.25</td>
<td>Biology 130 + 7.25 = 137.25</td>
</tr>
<tr>
<td>Math 10</td>
<td>Math 3.75</td>
<td>Math 10 + 3.75 = 13.75</td>
</tr>
<tr>
<td>Poetry 70</td>
<td>Poetry 5.32</td>
<td>Poetry 70 + 5.32 = 75.32</td>
</tr>
</tbody>
</table>

which says, for instance, that while the list price of the English textbook is $140, a shipping charge of $13.15 brings the price to be paid to $140 + $13.15 = $153.15.

3. Instead of representing the functions by tables, one might want to represent them by graphs. Rather than to use plots, though, one often uses bar graphs in which the pieces of input level lines that are between the 0-output level line and the plot point are highlighted into bars.

**Example 8.2.** The situation in the above example would be represented by the following bar graphs.

\[ x \xrightarrow{\text{LIST}} \text{LIST}(x) \quad x \xrightarrow{\text{SHIP}} \text{SHIP}(x) \quad x \xrightarrow{\text{PAY}} \text{PAY}(x) \]
8.2 Binomial Functions

1. Given a base function which is a monomial function, when we add-on a monomial function with the same exponent, the sum is a monomial function with the same exponent.

**Example 8.3.** Given the base function $MINT$ specified by the global input-output rule

\[
x \xrightarrow{MINT} MINT(x) = -12.82x^4
\]

and given the add-on function $TEA$ specified by the global input-output rule

\[
x \xrightarrow{TEA} TEA(x) = +49.28x^4
\]

then the sum function will be specified by the global input-output rule

\[
x \xrightarrow{SUM} SUM(x) = MINT(x) + TEA(x)
= -12.82x^4 \oplus +49.28x^4
= [-12.82 \oplus +49.28] x^4
= +36.46x^4
\]

2. However, when the exponent of the add-on function is different from the exponent of the base function, then the sum function is not a monomial function but what is called a binomial function.

**Example 8.4.** Let $BASE$ be specified by the global input-outut rule

\[
x \xrightarrow{BASE} BASE(x) = (-3)x^2
\]

and let $ADD-ON$ be specified by the global input-outut rule

\[
x \xrightarrow{ADD-ON} ADD-ON(x) = (+5)x^0
= +5
\]
then the $SUM$ function is specified by the global input-output rule

$$x \xrightarrow{SUM} SUM(x) = (-3)x^2 \oplus (+5)x^0$$

$$= (-3)x^2 + 5$$

To see that $SUM$ cannot be replaced by a single monomial function, we first evaluate all three functions at some input, for instance $+2$:

$$+2 \xrightarrow{BASE} BASE(+2) = (-3)(+2)^2$$

$$= -12$$

and

$$+2 \xrightarrow{ADD-ON} ADD-ON(+2) = (+5)(+2)^0$$

$$= +5$$

then

$$x \xrightarrow{SUM} SUM(x) = (-3)(+2)^2 \oplus (+5)(+2)^0$$

$$= -12 \oplus +5$$

$$= -7$$

The question then is what monomial function could return the output $-7$ for the input $+2$.

Of course, we can easily find a monomial function that would return the output $-7$ for the input $+2$. For instance, the dilation function $x \xrightarrow{f} f(x) = -\frac{7}{2}x$ does return the output $-7$ for the input $+2$. But $f$ is not going to return the same output as $SUM$ for other inputs, say, $+3$, $+4$, etc which it should. So, the binomial function

$$x \xrightarrow{SUM} SUM(x) = (-3)x^2 + 5$$

cannot be replaced by the single monomial function

$$x \xrightarrow{f} f(x) = -\frac{7}{2}x$$

**NOTE.** We noted at the beginning of Chapter 5 that monomial functions were only rarely called monomial functions and that this was unfortunate: indeed, it would be nicer to say that a binomial function cannot be replaced by a single monomial function. (We cannot have two for the price of one.)

### 8.3 Graphs of Binomial Functions

1. When the exponent of the add-on function is the same as the exponent of the base function, the bar graphs show exactly why the sum function will have again the same exponent.
8.3. Graphs of Binomial Functions

a. Given a constant base function, adding-on a constant function:

**Example 8.5.**

![Graphs](image)

b. Given a dilation base function, adding-on a dilation function:

**Example 8.6.**

![Graphs](image)

2. When the exponent of the add-on function is *not* the same as the exponent of the base function, the bar graphs show clearly why the sum function cannot be a monomial function.

a. Given a constant base function,

- Adding-on a dilation function:

**Example 8.7.**

![Graphs](image)

- Adding-on an even positive exponent monomial function:

**Example 8.8.**
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Example 8.9.

• Adding-on an odd positive exponent monomial function:

Example 8.10.

• Adding-on an even monomial function:

Example 8.11.

b. Given a dilation base function,
8.4 Trinomial Functions

There is of course no reason why the base function could not itself be a binomial function. In fact, this can very well be the case and the sum function will then be called a \textit{trinomial function}.

**Example 8.12.** Let $\text{BASE}$ be specified by the global input-output rule

$$x \xrightarrow{\text{BASE}} \text{BASE}(x) = (-3)x^0 \oplus (+7)x^1$$

and let $\text{ADD-ON}$ be specified by the global input-output rule

$$x \xrightarrow{\text{ADD-ON}} \text{ADD-ON}(x) = (+5)x^3$$

then the $\text{SUM}$ function is specified by the global input-output rule

$$x \xrightarrow{\text{SUM}} \text{SUM}(x) = (-3)x^0 \oplus (+7)x^1 \oplus (+5)x^3$$

$$= -3 + 7x + 5x^3$$

**Example 8.13.** Let $\text{BASE}$ be specified by the global input-output rule

$$x \xrightarrow{\text{BASE}} \text{BASE}(x) = (-3)x^1 \oplus (+7)x^0$$

and let $\text{ADD-ON}$ be specified by the global input-output rule

$$x \xrightarrow{\text{ADD-ON}} \text{ADD-ON}(x) = (+5)x^{-2}$$

then the $\text{SUM}$ function is specified by the global input-output rule

$$x \xrightarrow{\text{SUM}} \text{SUM}(x) = (-3)x^1 \oplus (+7)x^0 \oplus (+5)x^{-2}$$

$$= -3x + 7 + 5x^{-2}$$
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8.5 Comparing Monomial Functions
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