Chapter 5

Regular Power Functions -
Local Analysis

Power functions are functions that multiply or divide a given number by a number of copies of the input.

1. The output specifying code will be of the form:
   \[(\text{coefficient})\text{base}^\text{exponent}\]
   where:
   
   ▶ The \textit{coefficient} is the given number to be multiplied or divided by the copies of the input. The coefficient can be any \textit{bounded} signed number.
   
   ▶ The \textit{base} is the \textit{input} of which the copies are to be made.
   
   ▶ The \textit{exponent} is a \textit{signed counting} number which specifies what the function is to do to the \textit{coefficient} with the copies of the \textit{input}:
     
     • The \textit{size} of the exponent is the \textit{number of copies} of the input to be made. (If the exponent is 0, no copy is to be made and the coefficient is to be left alone.)
     
     • The \textit{sign} of the exponent specifies whether the coefficient is to be \textit{multiplied} or \textit{divided} by the copies of the input:
       
       + specifies that the coefficient is to be \textit{multiplied} by the copies of the input,
       
       – specifies that the coefficient is to be \textit{divided} by the copies of the input,
2. For reasons that will appear later, we will distinguish between:

- The regular power functions which are the power functions whose exponent is neither 0 nor +1, and which we will discuss in this chapter and in the following chapter, that is Chapter 6,

and

- The exceptional power functions which are the power functions whose exponent is either 0 or +1, and which we will discuss in Chapter 7.

### 5.1 Input-Output Links

Let \( f \) be the regular power function specified by the global input-output rule

\[
x \xrightarrow{f} f(x) = ax^{\pm n}
\]

and let \( x_0 \) be the specified input. Then, the procedure for getting \( f(x_0) \), namely the output returned by the function \( f \) for the specified input \( x_0 \), is:

**A. Output returned by \( f \) for the specified input \( x_0 \).**

1. **Declare** that \( x \) is to be replaced by [the specified input \( x_0 \)]

\[
x \xrightarrow{x \rightarrow x_0} f(x) \xrightarrow{x_0 \leftarrow x} ax^{\pm n}
\]

which, once the replacement has been carried out, gives:

\[
x_0 \xrightarrow{f} f(x_0) = ax_0^{\pm n}
\]

2. **Decode** the output specifying code:

   - If the exponent is positive, the code specifies that the output \( f(x_0) \) is obtained by multiplying the coefficient \( a \) by \( n \) copies of the specified input \( x_0 \):

\[
f(x_0) = a \cdot x_0 \cdot \ldots \cdot x_0
\]
5.1. INPUT-OUTPUT LINKS

• If the exponent is **negative**, the code specifies that the output $f(x_0)$ is obtained by **dividing** the coefficient $a$ by the $n$ copies of the specified input $x_0$:

$$f(x_0) = \frac{a}{x_0 \cdot \ldots \cdot x_0}$$

iii. **Perform** the computations specified by the code and thus get the output $y_0$.

iv. **Format** the **input-output link** according to the purpose:

- **Computational purpose**: $f(x_0) = y_0$
- **Functional purpose**: $x_0 \xrightarrow{f} y_0$
- **Graphic purpose**: $(x_0, y_0)$ is a plot-point for $f$

**EXAMPLE 1.** Let $FLIP$ be the function specified by the global input-output rule

$$x \xrightarrow{FLIP} FLIP(x) = (527.31)x^{+11}$$

(The function $FLIP$ is a regular power function.)

Let the specified input be $-3$. Then, to get the output returned by the function $FLIP$ for the specified input $-3$, we proceed as follows:

i. We **declare** that $x$ is to be replaced by the **specified input** $-3$.

$$x \xrightarrow{FLIP} FLIP(x) \bigg|_{x=-3} = (527.31)x^{+11} \bigg|_{x=-3}$$

which, once the replacement has been carried out, gives:

$$-3 \xrightarrow{FLIP} FLIP(-3) = (527.31) \cdot (-3)^{+11}$$

ii. We **decode** the output specifying code: since the exponent is **positive**, the code specifies that the output $FLIP(-3)$ is obtained by **multiplying** the coefficient $527.31$ by $11$ copies of the specified input $-3$:

$$FLIP(-3) = (527.31) \cdot (-3) \cdot \ldots \cdot (-3)$$

iii. We **perform** the computations specified by the code:
Separating the \textit{signs} from the \textit{sizes}, we have 
\[
= (527.31) \cdot \left(\begin{array}{c} 11 \text{ copies of } -3 \\ 11 \text{ copies of } 3 \end{array}\right) 
\]
and since

- by the \textbf{Sign Multiplication Rule}, \textit{any odd} number of copies of \textit{\(-3\)} multiply to \textit{\(-3\)}

\[
= (527.31) \cdot (-3) \cdot 1.77147 
= -93,411,384.57 
\]

\textbf{iv.} We format the \textit{input-output link} according to our purpose:

- For \textit{computational} purposes we would format the input-output link as: 

\[
\textit{FLOP}(-3) = -93,411,384.57 
\]

- For \textit{functional} purposes we would format the input-output link as: 

\[
-3 \xrightarrow{\text{FLOP}} -93,411,384.57 
\]

- For \textit{graphic} purposes we would format the input-output link as:

\[
(-3, -93,411,384.57) \text{ is a plot-point for FLOP} 
\]

\textbf{EXAMPLE 2.} Let \textit{FLOP} be the function specified by the global input-output rule 

\[
x \xrightarrow{\text{FLOP}} \text{FLOP}(x) = (+3,522.38)x^{-6} 
\]

(The function \textit{FLOP} is a \textit{regular} power function.)

Let the specified input be \textit{-3}. Then, to get the output returned by the function \textit{FLOP} for the specified input \textit{-3}, we proceed as follows:

\textbf{i.} We \textit{declare} that \textit{x} is to be replaced by \textit{the specified input -3} 

\[
x \bigg|_{x=-3} \xrightarrow{\text{FLOP}} \text{FLOP}(x) \bigg|_{x=-3} = (+3,522.38)x^{-6} \bigg|_{x=-3} 
\]

which, once the replacement has been carried out, gives:

\[
-3 \xrightarrow{\text{FLOP}} \text{FLOP}(-3) = (+3,522.38) \cdot (-3)^{-6} 
\]

\textbf{ii.} We \textit{decode} the output specifying code: since the exponent is \textit{negative}, the code specifies that the output \textit{FLOP}(-3) is obtained by \textit{dividing} the coefficient +3,522.38 by 6 copies of the specified input -3:

\[
\text{FLOP}(-3) = \frac{+3,522.38}{(-3) \cdot \ldots \cdot (-3)} 
\]

6 copies of \textit{-3}
5.2. TYPES OF REGULAR POWER FUNCTIONS

iii. We perform the computations specified by the code, Separating the signs from the sizes, we have

\[ = +3522.38 \]

\[ \text{(6 copies of -), (6 copies of +)} \]

and since,

- by the Sign Multiplication Rule, any even number of copies of \(-\) multiply to \(+\)

\[ = +3522.38 \]

\[ (+) \cdot 729 \]

\[ = +4831.796 \]

iv. We format the input-output link according to our purpose:

- For computational purposes we would format the input-output link as:

\[ FLOP(-3) = +4831.796 \]

- For functional purposes we would format the input-output link as:

\[ -3 \overset{FLOP}{\rightarrow} +4831.796 \]

- For graphic purposes we would format the input-output link as:

\[ (-3, +4831.796) \text{ is a plot-point for } FLOP \]

5.2 Types of Regular Power Functions

The features of the global input-output rule that specifies a regular power function come from the three numbers involved in the output specifying code.

1. Since, in this text, we will take a qualitative viewpoint, all the features of the global input-output rule will not be equally important for us. As we will see, the three features that will be important for us are:

- **Sign coefficient** which can be \(+\) or \(-\).
- **Sign exponent** which can be \(+\) or \(-\).
- **Parity exponent** which can be **even** or **odd** depending on whether the size of the exponent, that is the number of copies, is **even** or **odd**.
CHAPTER 5. LOCAL ANALYSIS

EXAMPLE 3. The function specified by the global input-output rule

\[ x \xrightarrow{\text{BLIP}} \text{BLIP}(x) = (-160.42)x^{7} \]

is a power function whose global input-output rule has the following features:
- Sign coefficient \( \text{BLIP} = - \).
- Sign exponent \( \text{BLIP} = + \).
- Parity exponent \( \text{BLIP} = \text{odd} \).

2. But, because, in this text, we are only interested in qualitative analysis, we will not pay any attention to the following two features:
- **Size coefficient** (other than the coefficient having to be bounded)
- **Size exponent**. (other than the size of the exponent being even or odd)

3. Accordingly, in order to focus on the important features of regular power functions, we will often **normalize** the global input-output rule of a regular power function as follows:

**B. To normalize the global I-O rule of a regular power function**

- **i.** We replace the size of the coefficient by the tag **bounded**.
- **ii.** We replace the size of the exponent by its parity, that is:
  - by the tag **even** when the size of the exponent is even
  - by the tag **odd** when the size of the exponent is odd

EXAMPLE 4. Let \( \text{BLIP} \) be the function specified by the global input-output rule

\[ x \xrightarrow{\text{BLIP}} \text{BLIP}(x) = (-160.42)x^{7} \]

In order to normalize \( \text{BLIP} \).

- **i.** We replace the size of the coefficient, namely 160.42, by the tag **bounded**.
- **ii.** We replace the size of the exponent, namely 7, by the tag **odd**.

The normalized global input-output rule of \( \text{BLIP} \) thus is

\[ x \xrightarrow{\text{BLIP}} \text{BLIP}(x) = (-\text{bounded}) \cdot x^{\text{odd}} \]

EXAMPLE 5. Let \( \text{BLOP} \) be the function specified by the global input-output rule

\[ x \xrightarrow{\text{BLOP}} \text{BLOP}(x) = (-365.28)x^{-6} \]

In order to normalize \( \text{BLOP} \),

- **i.** We replace the size of the coefficient, namely 365.28, by the tag **bounded**.
- **ii.** We replace the size of the exponent, namely 6, by the tag **even**.

The normalized global input-output rule of \( \text{BLOP} \) thus is

\[ x \xrightarrow{\text{BLOP}} \text{BLOP}(x) = (-\text{bounded}) \cdot x^{\text{even}} \]

4. Therefore, from the point of view of their global input-output rule, there will be **eight types** of regular power functions:
5.3. GRAPH PLACE NEAR $+\infty$

<table>
<thead>
<tr>
<th>Sign coefficient</th>
<th>Sign exponent</th>
<th>Parity exponent</th>
<th>Input-output rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>even</td>
<td>$(+\text{bounded})x^{+\text{even}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>odd</td>
<td>$(+\text{bounded})x^{+\text{odd}}$</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>even</td>
<td>$(+\text{bounded})x^{+\text{even}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>odd</td>
<td>$(+\text{bounded})x^{+\text{odd}}$</td>
</tr>
<tr>
<td>-</td>
<td>+</td>
<td>even</td>
<td>$(-\text{bounded})x^{-\text{even}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>odd</td>
<td>$(-\text{bounded})x^{-\text{odd}}$</td>
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<td></td>
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<td>even</td>
<td>$(-\text{bounded})x^{-\text{even}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>odd</td>
<td>$(-\text{bounded})x^{-\text{odd}}$</td>
</tr>
</tbody>
</table>

In what follows, we will discuss the way that the features of the global input-output rule that specify a regular function correspond to the local features of the qualitative global graph of that regular power function.

I. LOCAL ANALYSIS NEAR $\infty$

As we will see, getting the local graph near $\infty$ of a regular power function requires only that we locate the place of the local graph.

In addition to using the Sign Multiplication Rule and the Sign Division Rule, we will make a very heavy use of:

- The Definition of large and the Definition of small,
- The Size Multiplication Theorem and the Size Division Theorem.

5.3 Place of the Local Graph Near $+\infty$

1. First we want to make the case that the place of the local graph of a regular power function near $+\infty$ depends only on the sign of the coefficient and on the sign of the exponent.

First, we make the case that the local graph near $+\infty$ will be to the right of the screen no matter what.

Since the local graph near $+\infty$ is for inputs that are $+\text{large}$, the input-level lines show that the local graph near $+\infty$ will be somewhere in the following area no matter what:
ii. Next, we make the case that whether the local graph near $+\infty$ is above or below the 0-output level line depends only on the sign of the coefficient.

For positive inputs, the output will have the sign of the coefficient because, by the Sign Multiplication Rule, any number of copies of $+$ will multiply to $+$ so the sign of the output is the sign of the coefficient multiplied by $+$. Then, the output-level lines show that the local graph near $+\infty$ will be somewhere in either one of the following areas:

iii. Finally, we make the case that the place of the local graph near $\infty$ depends on the sign of the exponent:

- With a positive exponent, the coefficient will be multiplied by copies of large so that, by the Size Multiplication Theorem, the output will be large and the place of the local graph near $+\infty$ will be either one of the following depending on the sign of the coefficient.

- With a negative exponent, the coefficient will be divided by copies of large so that, by the Size Division Theorem, the output will be small and the place of the local graph near $+\infty$ will be either one of the following depending on the sign of the coefficient.
5.3. **GRAPH PLACE NEAR** $+\infty$

2. The procedure to get the *place* of the local graph near $+\infty$ for a regular power function is essentially the same as the procedure we used to link an input to an output. The difference is just that:

   a. **We normalize** the global input-output rule
   
   b. **We localize** the function near $+\infty$; that is, instead of declaring that $x$ is to be replaced by the *bounded input* $x_0$, we declare that $x$ is to be replaced by the *sign-size* $+\text{large}$.

So then, instead of returning an output number $y_0$, the function will return only the *sign-size* of the output and thus, instead of getting a *plot-point*, we will get the *place* of the local graph near $+\infty$.

**Example 6.** Let $NADE$ be the function specified by the global input-output rule

\[
  x \xrightarrow{NADE} NADE(x) = (-83.91)x^{-5}
\]

In order to find the *place* of the local graph near $+\infty$:

   a. We normalize $NADE$, that is
   
     i. We replace the *size* of the coefficient, namely $83.91$, by the tag *bounded*
   
     ii. We replace the *size* of the exponent, namely $5$, by the tag *odd*

The normalized global input-output rule of $NADE$ thus is

\[
  x \xrightarrow{NADE^*} NADE^*(x) = (-\text{bounded}) \cdot x^{-\text{odd}}
\]

   b. In order to **localize** $NADE$ near $+\infty$:

     i. We declare that $x$ is to be replaced by the *sign-size* $+\text{large}$.

   \[
   x \bigg|_{x \leftarrow +\text{large}} \xrightarrow{NADE} NADE(x) \bigg|_{x \leftarrow +\text{large}} = (-\text{bounded}) \cdot x^{-\text{odd}} \bigg|_{x \leftarrow +\text{large}}
   \]

     which, once carried out, gives:

     \[
     +\text{large} \xrightarrow{NADE} NADE(+\text{large}) = (-\text{bounded})(+\text{large})^{-\text{odd}}
     \]

     ii. We **decode** the output specifying code: since the exponent is *negative*, we get that the output $NADE(+\text{large})$ is obtained by *dividing* the coefficient $-\text{bounded}$ by an *odd* number of copies of the specified input $+\text{large}$:

     \[
     = \frac{-\text{bounded}}{(+\text{large}) \cdot \ldots \cdot (+\text{large})}
     \]

     odd number of copies of $+\text{large}$

     iii. We **perform** the computations specified by the code:
Separating the *signs* from the *sizes*, we have

\[
\begin{align*}
\text{role} & = -\text{bounded} \\
& = (+) \cdot \ldots \cdot (+) \cdot \text{large} \cdot \ldots \cdot \text{large}
\end{align*}
\]

and since,

- by the **Sign Multiplication Rule**, any number of copies of + multiply to +
- by the **Definition of large**, any number of copies of large multiply to large

\[
\begin{align*}
\text{role} & = -\text{bounded} \\
& = (+) \cdot \text{large}
\end{align*}
\]

Finally, by the **Size Division Theorem**, bounded divided by large is small and so we have

\[
\text{role} = \text{small}
\]

**iv.** Since our purpose is *graphic*, we format the input-output link as:

\[
(\text{+large, \text{small}})	ext{ is the place of the local graph near } \infty \text{ of NADE}
\]

c. We can now get the *place* of the local graph near \( +\infty \) of NADE

**EXAMPLE 7.** Let \( PADE \) be the function specified by the global input-output rule

\[
x \xrightarrow{\text{PADE}} PADE(x) = (-83.91)x^5
\]

In order to find the *place* of the local graph near \( +\infty \):

a. We normalize \( PADE \), that is

i. We replace the size of the coefficient, namely 83.91, by the tag *bounded*

ii. We replace the size of the exponent, namely 5, by the tag *odd*

The normalized global input-output rule of \( PADE \) thus is

\[
x \xrightarrow{\text{PADE}} PADE(x) = (-\text{bounded}) \cdot x^{+\text{odd}}
\]

b. In order to *localize* \( PADE \) near \( +\infty \):
5.3. GRAPH PLACE NEAR $+\infty$

i. We declare that $x$ is to be replaced by the sign-size $+\text{large}$,

\[
\begin{array}{c|c}
    x & x \leftarrow +\text{large} \\
{} & \xrightarrow{\text{PADE}} \\
\end{array}
\]

\[
\begin{array}{c|c}
    \text{PADE}(x) & (-\text{bounded}) \cdot x^{+\text{odd}} \\
{} & x \leftarrow +\text{large} \\
\end{array}
\]

which, once carried out, gives:

\[
+\text{large} \xrightarrow{\text{PADE}} \text{PADE}(+\text{large}) = (-\text{bounded})(+\text{large})^{+\text{odd}}
\]

ii. We decode the output specifying code: since the exponent is positive, we get that the output $\text{PADE}(+\text{large})$ is obtained by multiplying the coefficient $-\text{bounded}$ by an odd number of copies of the specified input $+\text{large}$:

\[
= (-\text{bounded}) \cdot (+\text{large}) \cdot \ldots \cdot (+\text{large})
\]

odd number of copies of $+\text{large}$

iii. We perform the computations specified by the code:

Separating the signs from the sizes, we have

\[
= (-\text{bounded}) \cdot (+) \cdot \ldots \cdot (+) \
\]

odd number of copies of $+$

and since,

- by the Sign Multiplication Rule, any number of copies of $+$ multiply to $+$
- by the Definition of large, any number of copies of large multiply to large

\[
= (-\text{bounded}) \cdot (+) \cdot \text{large}
\]

Finally, since, by the Size Multiplication Theorem, $\text{bounded} \cdot \text{large} = \text{large}$, we have

\[
= -\text{large}
\]

iv. Since our purpose is graphic, we format the input-output link as:

\[
(\text{+large}, -\text{large}) \text{ is the place of the local graph near } \infty \text{ of } \text{PADE}
\]

c. We can now get the place of the local graph near $+\infty$ of $\text{PADE}$
5.4 Place of the Local Graph Near $-\infty$

We get the place of the local graph near $-\infty$ exactly the same way as we get the place of the local graph near $+\infty$ but here, because the inputs are negative, we must take the parity of the exponent into consideration.

1. First we make the case that, in addition to depending on the sign of the coefficient and on the sign of the exponent, the place of the local graph near $-\infty$ depends also on the parity of the exponent.

   i. First we make the case that the local graph near $-\infty$ will be to the left of the screen no matter what.

   Since the local graph near $-\infty$ is for inputs that are large, the local graph near $-\infty$ will be somewhere in the following area:

   ![Diagram](image)

   ii. Next, we make the case that whether the local graph near $-\infty$ is above or below the 0-output level line depends not only on the sign of the coefficient as it did near $+\infty$, but also on the parity of the exponent. This is the big difference between what happens on the two sides of infinity, $+\infty$ and $-\infty$. Here is what happens near $-\infty$:

   - With an even exponent, the coefficient is multiplied or divided by an even number of copies of the negative input and by the Sign Multiplication Rule the product of these copies will be positive. So, the sign of the output will be the same as the sign of the coefficient. So the local place near $-\infty$ will be in either one of the following areas depending on the sign of the coefficient.

     ![Even exponent Diagram](image)

   - With an odd exponent, the coefficient is multiplied or divided by an odd number of copies of the negative input and by the Sign Multiplication Rule the product of these copies will be negative. So, the sign of the output will be the opposite of the sign of the coefficient. So the local place near $-\infty$ will be in either one of the following areas depending on the sign of the coefficient.

     ![Odd exponent Diagram](image)
5.4. GRAPH PLACE NEAR $-\infty$

- With an **odd exponent**, the coefficient is multiplied or divided by an odd number of copies of the negative input and by the **Sign Multiplication Rule** the product of these copies will be negative. So, the sign of the output will be the opposite of the sign of the coefficient. So the local place near $-\infty$ will be in either one of the following areas depending on the **sign of the coefficient**

In other words and to summarize:

**THEOREM 1** (Flip Theorem). For a regular power function, we get the local place near $-\infty$ by flipping the local place near $+\infty$ according to the **parity of the exponent**:

- **When the exponent is even**, we get the local place near $-\infty$ by flipping the local place near $+\infty$ **sideways**:

- **When the exponent is odd**, we get the local place near $-\infty$ by flipping the local place near $+\infty$ **diagonally**:
The case for the **Flip Theorem** is entirely based on the **Rule for Signs Multiplication**. As already discussed above, the **signs** of the copies of a **negative input** will multiply to

\( + \) when the **exponent is even**. So the sign of the output will be the **same as** the sign of the coefficient. So the place near \(-\infty\) will be on the **same side** of the 0-output level line as the place near \(+\infty\).

\( - \) when the **exponent is odd**. So the sign of the output will be the opposite of the sign of the coefficient. So the place near \(-\infty\) will be on the opposite side of the 0-output level line from the place near \(+\infty\).

iii. Finally, we make the case that the **place of the local graph near \(\infty\)** depends on the **sign of the exponent**:

- **Positive exponent**
  - With a **positive exponent**, the size of the coefficient will be multiplied by copies of **large** so that, by the **Size Multiplication Theorem**, the output will be **large** and the **place** of the local graph near \(-\infty\) will be either one of the following depending on the sign of the coefficient:

    - **Positive coefficient**
    - **Negative coefficient**

- **Negative exponent**
  - With a **negative exponent**, the size of the coefficient will be divided by copies of **large** so that, by the **Size Division Theorem**, the output will be **small** and the place of the local graph near \(-\infty\) will be either one of the following depending on the sign of the coefficient:

    - **Positive coefficient**
    - **Negative coefficient**

2. The procedure to get the **place** of the local graph near \(-\infty\) of a regular power function is exactly the same as the procedure we used to get the local graph near \(+\infty\) but here, with **negative inputs**, we must take the
5.4. GRAPH PLACE NEAR \(-\infty\)

Parity of the exponent into consideration.

**Example 8.** Let \(\text{NADE}\) be the function specified by the global input-output rule

\[
x \xrightarrow{\text{NADE}} \text{NADE}(x) = (-83.91)x^{-5}
\]

In order to find the place of the local graph near \(-\infty\):

- We normalize \(\text{NADE}\), that is
  - We replace the size of the coefficient, namely 83.91, by the tag \(\text{bounded}\)
  - We replace the size of the exponent, namely 5, by the tag \(\text{odd}\)

The normalized global input-output rule of \(\text{NADE}\) thus is

\[
x \xrightarrow{\text{NADE}} \text{NADE}(x) = (-\text{bounded}) \cdot x^{-\text{odd}}
\]

- In order to localize \(\text{NADE}\) near \(-\infty\):
  - We declare that \(x\) is to be replaced by the sign-size \(-\text{large}\)

\[
x \xleftarrow{x \leftarrow \text{large}} \xrightarrow{\text{NADE}} \text{NADE}(x) \xleftarrow{x \leftarrow \text{large}} \xrightarrow{(-\text{large}) \cdot \ldots \cdot (-\text{large})} \text{NADE}(-\text{large}) = (-\text{bounded})(-\text{large})^{-\text{odd}}
\]

  - We decode the output specifying code: since the exponent is \(\text{negative}\), we get that the output \(\text{NADE}(-\text{large})\) is obtained by dividing the coefficient \(-\text{bounded}\) by an odd number of copies of the specified input \(-\text{large}\):

    \[
    = (-\text{bounded}) \cdot \ldots \cdot (-\text{large})
    \]

  - We perform the computations specified by the code:
    - Separating the signs from the sizes, we have

      \[
      = (-\text{bounded}) \cdot \ldots \cdot (-\text{large}) \\
      \text{odd number of copies of } -\text{large}
      \]

\[
\text{odd number of copies of } \text{large}
\]
and since,

- by the **Sign Multiplication Rule**, an *odd* number of copies of -- multiply to --
- by the **Definition of large**, any number of copies of *large* multiply to *large*

\[
\begin{align*}
-\text{bounded} \\
\times \text{large}
\end{align*}
\]

Finally, by the **Size Division Theorem**, *bounded* divided by *large* is *small* and so we have

\[
\begin{align*}
-\text{small}
\end{align*}
\]

**iv.** Since our purpose is *graphic*, we format the input-output link as:

\[
(-\text{large}, +\text{small}) \text{ is the place of the local graph near } \infty \text{ of } \text{NADE}
\]

**c.** We can now get the place of the local graph near \(-\infty\) of \text{NADE}

\[
\begin{array}{c}
\text{Output Ruler} \\
\begin{array}{c}
0 \\
-\infty
\end{array}
\end{array}
\begin{array}{c}
\text{Input Ruler} \\
\begin{array}{c}
0 \\
+\infty
\end{array}
\end{array}
\begin{array}{c}
\text{Screen} \\
\text{Place of the Local Graph near } -\infty
\end{array}
\begin{array}{c}
\text{Ruler} \\
\begin{array}{c}
-\text{large}, +\text{small}
\end{array}
\end{array}
\begin{array}{c}
\text{Offscreen}
\end{array}
\]

**EXAMPLE 9.** Let \textit{PADE} be the function specified by the global input-output rule

\[
x \xrightarrow{\text{PADE}} \text{PADE}(x) = (-83.91)x + 5
\]

In order to find the place of the local graph near \(-\infty\):

**a.** We normalize \textit{PADE}, that is

\[
\begin{array}{c}
i. \text{ We replace the size of the coefficient, namely } 83.91, \text{ by the tag } \text{bounded} \\
ii. \text{ We replace the size of the exponent, namely } 5, \text{ by the tag } \text{odd}
\end{array}
\]

The normalized global input-output rule of \textit{PADE} thus is

\[
x \xrightarrow{\text{PADE}} \text{PADE}(x) = (-\text{bounded}) \cdot x^{+\text{odd}}
\]

**b.** In order to localize \textit{PADE} near \(-\infty\):

\[
\begin{array}{c}
i. \text{ We declare that } x \text{ is to be replaced by the sign-size } -\text{large} \\
\end{array}
\]

\[
x \bigg|_{x=\text{large}} \xrightarrow{\text{PADE}} \text{PADE}(x) \bigg|_{x=\text{large}} = (-\text{bounded})x^{+\text{odd}} \bigg|_{x=\text{large}}
\]
5.4. **GRAPH PLACE NEAR** $-\infty$

which, once carried out, gives:

$$\text{large} \xrightarrow{\text{PADE}} \text{PADE}(\text{large}) = (-\text{bounded})(\text{large})^{\text{odd}}$$

ii. We *decode* the output specifying code: since the exponent is positive, we get that the output $\text{PADE}(\text{large})$ is obtained by **multiplying** the coefficient $-\text{bounded}$ by an **odd** number of copies of the specified input $\text{large}$:

$$= (-\text{bounded}) \underbrace{\text{large} \cdot \ldots \cdot \text{large}}_{\text{odd number of copies of large}}$$

iii. We *perform* the computations specified by the code:

Separating the **signs** from the **sizes**, we have

$$= (-\text{bounded}) \cdot \underbrace{\text{large} \cdot \ldots \cdot \text{large}}_{\text{odd number of copies of large}}$$

and since,

- by the **Sign Multiplication Rule**, an **odd** number of copies of $-$ multiply to $-$
- by the **Definition of large**, any number of copies of large multiply to large

Finally, by the **Size Multiplication Theorem**, bounded multiplied by large is large and so we have

$$= +\text{large}$$

iv. Since our purpose is graphic, we format the *input-output link* as:

$(-\text{large}, +\text{large})$ is the place of the local graph near $\infty$ of $\text{PADE}$

c. We can now get the place of the local graph near $-\infty$ of $\text{PADE}$
5.5 Local Graph Near $\infty$

We finally come to how to get what we will need most of the time, that is the local graph of regular functions on both sides of $\infty$.

1. We could of course begin by getting separately the place of the local graph near $+\infty$ and the place of the local graph near $-\infty$. In practice, though, because we will need to get the local graph of regular functions near $\infty$ very quickly, we will get both places at the same time. The procedure will again be essentially the same:
   a. We normalize the global input-output rule
   b. We localize the function near $\infty$ that is, instead of declaring that $x$ is to be replaced by the sign-size $+\infty$ or by the sign-size $-\infty$, we declare that $x$ is to be replaced by the sign-size $\pm \text{large}$.

**Example 10.** Let $\text{NADE}$ be the function specified by the global input-output rule

$$x \xrightarrow{\text{NADE}} \text{NADE}(x) = (-83.91)x^{-5}$$

In order to find the place of the local graph near $\pm \infty$:
   a. We normalize $\text{NADE}$, that is
      i. We replace the size of the coefficient, namely 83.91, by the tag bounded
      ii. We replace the size of the exponent, namely 5, by the tag odd

The normalized global input-output rule of $\text{NADE}$ thus is

$$x \xrightarrow{\text{NADE}} \text{NADE}(x) = (-\text{bounded}) \cdot x^{-\text{odd}}$$

b. In order to localize $\text{NADE}$ near $\pm \infty$:
   i. We declare that $x$ is to be replaced by the sign-size $\pm \text{large}$

   $$x \xrightarrow{x \to \pm \text{large}} \text{NADE}(x) \xrightarrow{x \to \pm \text{large}} = (-\text{bounded}) \cdot x^{-\text{odd}}$$

   which, once carried out, gives:

   $$\pm \text{large} \xrightarrow{\text{NADE}} \text{NADE}(\pm \text{large}) = (-\text{bounded})(\pm \text{large})^{-\text{odd}}$$

   ii. We decode the output specifying code: since the exponent is negative, we get that the output $\text{NADE}(\pm \text{large})$ is obtained by dividing the coefficient $-\text{bounded}$ by an odd number of copies of the specified input $\pm \text{large}$.

   $$= \frac{-\text{bounded}}{(\pm \text{large}) \cdots (\pm \text{large})^{\text{odd number of copies of } \pm \text{large}}}$$
iii. We perform the computations specified by the code:
Separating the \textit{signs} from the \textit{sizes}, we have
\[
-\text{bounded} = (-\pm \ldots -\pm) \cdot (\pm \large \ldots \pm) \\
\text{odd number of copies of } \pm \text{ \quad odd number of copies of } \large
\]
and since,
\begin{itemize}
  \item by the \textbf{Sign Multiplication Rule}, any number of copies of + multiply to + and any odd number of copies of – multiply to –
  \item by the \textbf{Definition} of large, any number of copies of large multiply to large
\end{itemize}
Finally, by the \textbf{Size Division Theorem}, bounded divided by large is small and so we have
\[
+\text{small}
\]
iv. Since our purpose is graphic, we format the input-output link as:
\[
(+\text{large}, +\text{small}) \text{ is the place of the local graph near } \infty \text{ of } NADE
\]
c. We can now get the place of the local graph near $\pm\infty$ of $NADE$

2. In the case of a regular power function, the \textit{shape} of the local graph near $\infty$ is forced by the \textit{place}.

\textbf{THEOREM 2 (Shape Near }$\infty$\textbf{). As inputs get nearer to }$\infty$\textbf{, that is as inputs get larger in size:}
\begin{itemize}
  \item Regular power functions with \textbf{positive exponents} \textit{“straightline”} vertically:
\end{itemize}
3. Making the case for the Shape Near $\infty$ Theorem on the basis of the global input-output rule is not difficult but is a bit longish so we will not do it here. On the other hand, it is easy to see that the shape of the local graph near $\infty$ could not be any other than what the Shape Near $\infty$ Theorem says.
Example 11. Given the place

i. The slope cannot be \( \backslash \) as in

because, as inputs get larger, outputs would get smaller while the definition of large says that outputs have to get larger. So, the slope has to be /.

ii. The concavity cannot be \( \cap \) as in

because, as inputs get larger, outputs would eventually cease to get larger. So, the concavity has to be \( \cup \).

Altogether then, since the slope has to be / and the concavity has to be \( \cup \), the shape of the local graph near \(+\infty\) can only be:

II. Local Analysis Near \( 0 \)

The computations for getting the place of the local graph near \( \infty \) will use the following from Chapter 2:

- Rule for Sign Multiplication and Division
- Definition of large and small.
- Size Multiplication Theorem and Size Division Theorem as a result of which we immediately have the extremely important

**Theorem 3 (\( \infty \)-Height).** For positive-exponent power functions, all bounded inputs have bounded height and 0 is a 0-height input.
For negative-exponent power functions, 0 is an $\infty$-height input, that is nearly inputs have $\infty$ height.

**Example 12.** Given the function $RHON$ specified by the global input-output rule

$$x \xrightarrow{RHON} RHON(x) = (+93.29) \cdot x^3$$

we have:

$\text{bounded } \xrightarrow{RHON} RHON(\text{bounded}) = (+93.29) \cdot \text{bounded}^3$

$$= (+93.29) \cdot \text{bounded} \cdot \text{bounded} \cdot \text{bounded}$$

and, by the **Size Multiplication Theorem**

$$= \text{bounded}$$

In particular,

$$\text{small } \xrightarrow{RHON} RHON(\text{small}) = (+93.29) \cdot \text{small}^3$$

$$= (+93.29) \cdot \text{small} \cdot \text{small} \cdot \text{small}$$

and, by the **Size Multiplication Theorem**

$$= \text{small}$$

so that:

$$0 \xrightarrow{RHON} RHON(0) = 0$$

**Example 13.** Given the function $DION$ specified by the global input-output rule

$$x \xrightarrow{DION} DION(x) = (+13.72) \cdot x^{-4}$$

we have:

$\text{bounded } \xrightarrow{DION} DION(\text{bounded}) = (+13.72) \cdot \text{bounded}^{-4}$

$$= \frac{+13.72}{\text{bounded} \cdot \text{bounded} \cdot \text{bounded} \cdot \text{bounded}}$$

$$= \text{bounded}$$

and, by the **Size Division Theorem**

$$= \text{bounded}$$

However,

$$\text{small } \xrightarrow{DION} DION(\text{small}) = (+13.72) \cdot \text{small}^{-4}$$

$$= \frac{+13.72}{\text{small} \cdot \text{small} \cdot \text{small} \cdot \text{small}}$$

$$= \text{small}$$

and, by the **Size Division Theorem**

$$= \text{large}$$

so that:

$$0 \xrightarrow{DION} DION(0) = \infty$$
5.5. **LOCAL GRAPH NEAR $\infty$**

1. If all we want is the local graph near $-\infty$, we proceed exactly in the same manner as for the local graph near $+\infty$.

**Example 14.** Given the function specified by the **global input-output rule**

$$x \xrightarrow{KATE} KATE(x) = (-13.14) \cdot x^{-24}$$

in order to find the place of the local graph of $KATE$ near $-\infty$:

i. We **normalize** the global input output rule:

$$x \xrightarrow{KATE} KATE(x) = (\text{bounded}) \cdot x^{-\text{even}}$$

ii. To **localize** near $-\infty$ we compute the output for inputs that are $-\text{large}$:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\xrightarrow{KATE} KATE(x)$</th>
<th>$+$large</th>
<th>$-large$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\text{large}$</td>
<td>$(-\text{bounded})^{-\text{even}}$</td>
<td>$-\text{large}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(\text{bounded})\cdot(-\text{large})^{-\text{even}}$</td>
<td>$-\text{bounded}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(\text{large})\cdot\ldots\cdot(\text{large})$</td>
<td>$\text{even number of copies of } -\text{large}$</td>
<td></td>
</tr>
</tbody>
</table>

Separating the signs from the sizes, we have

$$= (-\text{bounded}) \cdot \ldots \cdot (-) \cdot (\text{large}) \cdot \ldots \cdot (\text{large})$$

and since

- by the **Sign Multiplication Rule**, an even number of copies of $-$ multiply to $+$
- by the **Definition of large**, any number of copies of large multiply to large

$$= -\text{bounded} \cdot (+) \cdot (\text{large})$$

Finally, by the **Size Division Theorem**, bounded divided by large is small and we have

$$= -\text{small}$$

iii. And so we have that:

$KATE(-\text{large}) = -\text{small}$

and that the place of the local graph of $KATE$ near $-\infty$ is:

2. **Example 15.** Given the function specified by the global input-output rule

$$x \xrightarrow{DAVE} DAVE(x) = (-83.17)x^{-13}$$
in order to find the place of the local graph of DAVE near $\infty$

i. We get the place of the local graph near $+\infty$ by computing from the global input-output rule as we did above:

ii. We get the place of the local graph near $-\infty$, by flipping the local graph near $+\infty$ according to the Place Near $-\infty$ Theorem:

iii. Altogether then, the place of the local graph near $\infty$ is:

3. **5.6 Place of the Local Graph Near $0^+$**

Since the local graph near $0^+$ is for $+small$ inputs, the local graph near $0^+$ will be somewhere in the following area

so that is the local graph near $0^+$ will be onscreen or offscreen depending on the size of the outputs.

1. More precisely, since the inputs are positive small, i. We get the sign of the output from the sign of the coefficient since the inputs are positive and, by the Rule for Sign Multiplication, any number
of copies of $+$ will multiply to $+$. So the local graph near $0^+$ will be, depending on the sign of the coefficient, either one of the following:

ii. We get the size of the output from the sign of the exponent:

- If the exponent is positive, the coefficient will be multiplied by the copies of the small input so that, by the Size Multiplication Theorem, the output will be small and the place of the local graph near $0^+$ will be, depending on the sign of the coefficient, either one of the following:

- If the exponent is negative, the coefficient will be divided by the copies of the small input so that, by the Size Division Theorem, the output will be large and the place of the local graph near $+\infty$ will be, depending on the sign of the coefficient, either one of the following:

2. In practice, though, we will deal at the same time with both the size
CHAPTER 5. LOCAL ANALYSIS

and the sign of the inputs.

**EXAMPLE 16.** Given the function specified by the *global input-output rule*
\[ x \xrightarrow{KATE} KATE(x) = (-13.14) \cdot x^{+24} \]
in order to find the place of the local graph near 0⁺:

i. We *normalize* the global input output rule:
\[ x \xrightarrow{KATE} KATE(x) = (-1) \cdot x^{+\text{even}} \]

ii. We *compute* the output for inputs that are \(+\text{small}\):

\[
\begin{array}{c|c}
 x & x \xrightarrow{KATE} KATE(x) \\
 \hline
 +\text{small} & (-1) \cdot (+\text{small})^{+\text{even}} \\
 & = (-1) \cdot (+\text{small}) \cdot \ldots \cdot (+\text{small}) \\
 & \quad \text{even number of copies of } +\text{small} \\
 & = (-1) \cdot \underbrace{(+) \cdot \ldots \cdot (+)}_{\text{even number of copies of } +} \cdot \underbrace{(\text{small}) \cdot \ldots \cdot (\text{small})}_{\text{even number of copies of } \text{small}} \\
\end{array}
\]

and since by the *Rule for Sign Multiplication*, any number of copies of + multiply to +
\[ = (-1) \cdot (+) \cdot \underbrace{(\text{small}) \cdot \ldots \cdot (\text{small})}_{\text{even number of copies of } \text{small}} \]

and since, by the *Definition of small*, any number of copies of small multiply to small
\[ = (-1) \cdot ((+) \cdot \text{small}) \]

and since, by the *Size Multiplication Theorem*, bounded multiplied by small is small
\[ = -\text{small} \]

iii. And so we have that:
\[ KATE(+\text{small}) = -\text{small} \]
and that the *place* of the local graph of KATE near 0⁺ is:

**EXAMPLE 17.** Given the function specified by the *global input-output rule*
\[ x \xrightarrow{DATE} DATE(x) = (-13.14) \cdot x^{-8} \]
in order to find the place of the local graph near 0⁺:
5.7. GRAPH PLACE NEAR 0^−

i. We normalize the global input output rule:

\[ x \xrightarrow{DATE} DATE(x) = (-1) \cdot x^{-\text{even}} \]

ii. We compute the output for inputs that are +small:

\[
\begin{array}{c|c|c|c}
\hline
x & x:=+small & DATE(x) & x:=+small \\
\hline
& & = (-1)x^{-\text{even}} & \\
& & = (-1) \cdot (+\text{small})^{-\text{even}} & \\
& & = -1 & \\
& & = (+\text{small}) \cdot \ldots \cdot (+\text{small}) & \\
& & \text{even number of copies of +small} & \\
& & = -1 & \\
& & = (+) \cdot \ldots \cdot (+) \cdot (\text{small}) \cdot \ldots \cdot (\text{small}) & \\
& & \text{even number of copies of +} \cdot \text{even number of copies of small} & \\
\hline
\end{array}
\]

and since, by the Rule for Sign Multiplication, any number of copies of + multiply to +

\[ = -1 \]

and since, by the Definition of small, any number of copies of small multiply to small

\[ = -1 \cdot (\text{small}) \]

and since, by the Size Division Theorem, bounded divided by small is small

\[ = -\text{large} \]

iii. And so we have that:

\[ DATE(+\text{small}) = -\text{large} \]

and that the place of the local graph of DATE near 0^+ is:

\[ \begin{array}{c}
\text{Output Ruler} \\
\text{Screen} \\
-\infty \\
\hline
\text{Input Ruler} \\
0 \\
0^+ \\
\end{array} \]

5.7 Place of the Local Graph Near 0^−

Since the local graph near 0^− is for − small inputs, the local graph near 0^− will be somewhere in the following area
that is the local graph near $0^-$ will be onscreen or offscreen depending on the size of the outputs.

1. If all we want is the local graph near $0^-$, we proceed exactly in the same manner as for the local graph near $0^+$.

**Example 18.** Given the function specified by the global input-output rule

$$x \xrightarrow{DATE} DATE(x) = (-13.14) \cdot x^{-24}$$

in order to find the place of the local graph near $0^-$:

i. We normalize the global input output rule:

$$x \xrightarrow{DATE} DATE(x) = (-1) \cdot x^{\text{even}}$$

ii. We compute the output for inputs that are $-\text{small}$:

\[
\begin{array}{c|c}
\hline
x & DATE(x) \\
\hline
-\text{small} & (-1) \cdot (-\text{small})^{\text{even}} \\
& = (-1) \cdot (\text{small})^{\text{even}} \\
& = 1 \\
& \text{even number of copies of } -\text{small} \\
& \text{even number of copies of } \text{small} \\
\hline
\end{array}
\]

and since, by the **Rule for Sign Multiplication**, an even number of copies of $-$ multiplies to $+$

$$= -1$$

$$= (+) \cdot (\text{small})^{\text{even}} \cdot (\text{small})^{\text{even}}$$

and since, by the **Definition of small**, copies of $\text{small}$ multiply to $\text{small}$

$$= -1$$

$$= (+) \cdot \text{small}$$
and since, by the **Size Division Theorem**, bounded divided by small is large

= \text{large}

\(0^{-29}\)

\[\]

iii. And so we have that:
\(DATE(-small) = -large\)

and that the place of the local graph of \(DATE\) near \(0^-\) is:

\[\]

2. If we want the local graph near \(0^-\) as part of the local graph near \(0\), after we have gotten the local graph near \(+\infty\) we obtain the local graph near \(0^-\) from the local graph near \(0^+\) by the following

**THEOREM 4 (Place Near \(0^-\)).** For a regular power function, the local place near \(0^-\) is obtained by flipping the local place near \(0^+\) according to the parity of the exponent:

- **When the exponent is even**, the local place near \(0^-\) is obtained by flipping the local graph near \(0^+\) horizontally:

  Positive exponent:
  
  ![Positive exponent diagram](image)

  Negative exponent:
  
  ![Negative exponent diagram](image)

- **When the exponent is odd**, the local place near \(0^-\) is obtained by flipping the local graph near \(0^+\) diagonally:

  Positive exponent:
  
  ![Positive exponent diagram](image)

  Negative exponent:
  
  ![Negative exponent diagram](image)
EXAMPLE 19. Given the function specified by the global input-output rule

\[ x \xrightarrow{DATE} DATE(x) = (-13.14) \cdot x^{-24} \]

in order to find the place of the local graph of \( DATE \) near 0

i. We get the place of the local graph near \( 0^+ \) by computing from the global input-output rule as we did above:

ii. We get the place of the local graph near \( 0^- \), by flipping the local graph near \( 0^+ \) according to the Place Near \( 0^- \) Theorem:

iii. Altogether then, the place of the local graph near 0 is:

The case for the Place Near \( 0^- \) Theorem goes exactly the same way as the case for the Place Near \( -\infty \) Theorem that is it is entirely based on the Rule for Signs Multiplication:

- When Parity exponent = even, multiplying the signs of the copies of the input will give +. The sign of the output will then be the sign of the coefficient multiplied by + and thus will be the same as the sign of the coefficient. So the place near \( -\infty \) will be on the same side of 0 as the place near \( +\infty \),
- When Parity exponent = odd, multiplying the signs of the copies of the input will give -. The sign of the output will then be the sign of the coefficient multiplied by + and thus will be the opposite of the sign of the coefficient. So the place near \( -\infty \) will be on the other side of 0 from the place near \( +\infty \).
5.8 Shape of the Local Graph Near 0

The shape of the local graph of a regular power function near 0 is forced by its place.

1. More precisely:

**THEOREM 5 (Shape Near 0).** As inputs get nearer to 0:

- Positive exponents power functions “flatline” horizontally:

- Negative exponents power functions “flatline” vertically:

The local graphs near 0 for regular power functions with positive exponents flatline horizontally as they get nearer to 0:

The local graphs near 0 for regular power functions with negative exponents flatline vertically as they get nearer to 0:
2. Making the case for the **Shape Near 0 Theorem** computationally is not difficult but is a bit longish so we will not do it here. On the other hand, it is easy to see that the shape of the local graph near 0 could not be any other than what the **Shape Near 0 Theorem** says.

**Example 20.** Given the place

i. The slope cannot be $\downarrow$ as in because, as inputs get smaller, outputs would get smaller while the global input-output rule says that outputs have to get larger. So, the slope has to be $\uparrow$.

ii. The concavity cannot be $\cup$ as in because, as inputs get smaller, outputs would eventually cease to get larger. So, the concavity has to be $\cap$.

Altogether then, since the slope has to be $\uparrow$ and the concavity has to be $\cap$, the shape of the local graph near $0^+$ can only be:

3. In practice, to get the local graph near 0 we:
5.8. GRAPH SHAPE NEAR 0

i. Compute from the global input-output rule the place of the local graph near 0).

ii. Get the place of the graph near 0− by flipping the place of the graph near 0+ according to the Place Near 0− Theorem.

iii. Get the shape of the graph near 0 by the Shape Near 0 Theorem.

In other words, to get the local graph near 0, we proceed as just as we did to get the local graph near ∞.

Example 21. Given the function specified by the global input-output rule

\[ x \xrightarrow{KATE} KATE(x) = (-13.14) \cdot x^{+24} \]

in order to find the local graph of KATE near 0.

i. We get the place of the local graph near 0+

from the global input-output rule by computing as we did above:

ii. We get the place of the local graph near 0−, by flipping the local graph near 0+ according to the Place Near 0− Theorem:

iii. Altogether then, the place of the local graph near 0 is:
iv. We get the shape of the local graph near 0 by the Shape Near 0 Theorem

**Example 22.** Given the function specified by the global input-output rule

\[
x \xrightarrow{DAVE} DAVE(x) = (+83.17)x^{+13}
\]

in order to find the local graph of \( DAVE \) near 0.

i. We get the place of the local graph near \( 0^+ \) by computing from the global input-output rule as we did above:

ii. We get the place of the local graph near \( 0^- \), by flipping the local graph near \( 0^+ \) according to the Place Near 0^- Theorem:

iii. Altogether then, the place of the local graph near 0 is:
iv. We get the shape of the local graph near 0 by the **Shape Near 0 Theorem**
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