Chapter 10

Affine Problems

10.1 Introduction

The most frequent type of real-world situations is where we want to find the situations in which the money worth of a collection plus some fixed money amount compares in a given way with a given gauge.

1. The corresponding problem is called an affine problem and we shall also use the terms affine formula, affine equation and affine inequation.

The number-phrase that represents the fixed money amount is called the constant term.

**Example 1.** Jane wants to buy three apples but there is a fixed transaction charge of four dollars and fifty cents and the most she wants to spend is twenty-three dollars and thirty-four cents. So, whether or not she will be able to get the three apples will depend on the on the going unit-worth of the apples.

The real-world situation is represented by the inequation

\[3 \text{ Apples } \times x \frac{\text{ Dollars}}{\text{ Apple}} + 4.5 \text{ Dollars} \leq 23.34 \text{ Dollars}\]

where 4.5 Dollars is the constant term.

When we carry out the co-multiplication we get the affine inequation

\[3 \times x + 4.5 \text{ Dollars} \leq 23.34 \text{ Dollars}\]

\[[3 \times x] \text{ Dollars} + 4.5 \text{ Dollars} \leq 23.34 \text{ Dollars}\]

When we factor out the common denominator Dollars, we get the affine problem in Dollars

\[3 \times x + 4.5 \leq 23.34\]

2. *Translation* problems and *dilation* problems as well as *basic* problems
turn out to be special cases of affine problems which are therefore a more general type of problems:

- If the number of items in an affine problem is 1, then the affine problem is really just a translation problem.

**Example 2.** If the number of items in Example 1 were 1 instead of 3, then the inequation would be
\[
1 \text{ Apples} \times x \text{ Dollars} + 4.5 \text{ Dollars} \leq 23.34 \text{ Dollars}
\]
which boils down to the inequation in Dollars
\[
x + 4.5 \leq 23.34
\]
which is a translation problem.

- If the fixed number-phrase in an affine problem is 0, then that affine problem is really just a dilation problem.

**Example 3.** If the fixed number-phrase in Example 1 were 0 Dollars instead of 4.5 Dollars, then the inequation would be
\[
3 \text{ Apples} \times x \text{ Dollars} + 0 \text{ Dollars} \leq 23.35 \text{ Dollars}
\]
which boils down to the inequation in Dollars
\[
3x \leq 23.35
\]
which is a dilation problem.

- If, in an affine problem, both the additional number-phrase is 0 and the number of items is 1, then that affine problem is really just a basic problem.

**Example 4.** If, in Example 24 the number of items were 1 instead of 3 and the additional number-phrase were 0 Dollars instead of 4.5 Dollars, then the inequation would be
\[
1 \text{ Apples} \times x \text{ Dollars} + 0 \text{ Dollars} \leq 23.35 \text{ Dollars}
\]
which boils down to the inequation in Dollars
\[
x \leq 23.35
\]
which is a basic problem.

### 10.2 Solving Affine Problems

We now turn to the investigation of the solution subset of affine problems which we will do in accordance with the general procedure. The investigation of affine problems will proceed much in the same way as that of translation and dilation problems. As usual, the only difficulty will be that, although similar in nature, problems may involve numerators of different kinds:

- plain counting numerators to represent numbers of items,
- signed counting numerators to represent two-way numbers of items,
- plain decimal numerators to represent quantities of stuff,
- signed decimal numerators to represent two-way quantities of stuff.
1. We locate the boundary point of the solution subset. This involves the following steps:

i. We write the associated equation for the given problem:

**EXAMPLE 5.** Given the affine problem in Dollars in **EXAMPLE 1**
\[ 3 \times x + 4.5 \leq 23.34 \]

the associated equation in Dollars is
\[ 3 \times x + 4.5 = 23.34 \]

ii. We try to solve the associated equation in two stages by way of the Reduction Approach:

   i. We try to reduce the affine problem to a dilation problem by subtracting the fixed term from both sides so as to be able to invoke the Fairness Theorem,

   ii. We then try to reduce the resulting dilation problem to a basic problem by dividing by the coefficient of \( x \) both sides so as to be able to invoke the Fairness Theorem,

**EXAMPLE 6.** Given the affine equation in Dollars in **EXAMPLE 2**
\[ 3 \times x + 4.5 = 23.34 \]

   i. We subtract 4.5 from both sides:
\[ 3 \times x + 4.5 - 4.5 = 23.34 - 4.5 \]

which boils down to the dilation equation in Dollars
\[ 3 \times x = 18.84 \]

   ii. We divide both sides by 3
\[ 3 \times x \div 3 = 18.84 \div 3 \]

which boils down to the basic equation in Dollars
\[ x = 6.28 \]

2. We locate the interior of the solution subset according to the General Procedure. (For the sake of completion, we include in the **EXAMPLE** the step in which we get the boundary point.

**EXAMPLE 7.** Given the affine problem in Dollars in **EXAMPLE 1**:
\[ 3 \times x + 4.5 \leq 23.34 \]

i. To get the boundary of the solution subset

   i. We locate the boundary point as in **EXAMPLE 6**: 6.28

   ii. Since the inequation is lenient, the boundary point is included in the solution subset and so we graph it with a solid dot.

   ii. We locate the interior of the solution subset

   i. The boundary point 6.28 Dollars divides the data set into two sections:
ii. We test Section A with, for instance, 0.1 and, since
$$3 \times x + 4.5 \leq 23.34 \bigg|_{x=0.1}$$
we get that 0.1 is a solution of the inequation in Dollars
$$3 \times x + 4.5 \leq 23.34$$
and Pasch’s Theorem then tells us that all number-phrases in Section A are included in the solution subset.

iii. We test Section B with, for instance, +5.0 and, since
$$3 \times x + 4.5 \leq 23.34 \bigg|_{x=1000}$$
we get that 1000 is a non-solution of the inequation in Dollars
$$3 \times x + 4.5 \leq 23.34$$
and Pasch’s Theorem then tells us that all number-phrases in Section B are non-included in the solution subset.

iii. Altogether, we represent the solution subset of the inequation in Dollars
$$3 \times x + 4.5 \leq 23.34$$
as follows:
- The graph of the solution subset is
- The name of the solution subset is
$$(0, 6.28)$$