problem, affine formula, affine equation, affine inequation, affine term, constant

Chapter 10

Affine Problems

10.1 Introduction

The most frequent type of real-world situations is where we want to find the situations in which the money worth of a collection *plus some fixed money amount* compares in a given way with a given gauge.

1. The corresponding problem is called an **affine problem** and we shall also use the terms **affine formula**, **affine equation** and **affine inequation**.

The number-phrase that represents the fixed money amount is called the **constant term**.

EXAMPLE 1. Jane wants to buy three apples but there is a fixed transaction charge of four dollars and fifty cents and the most she wants to spend is twenty-three dollars and thirty-four cents. So, whether or not she will be able to get the three apples will depend on the on the going *unit-worth* of the apples.

The real-world situation is represented by the inequation

$$3 \text{ Apples} \times x \frac{\text{Dollars}}{\text{Apple}} + 4.5 \text{ Dollars} \leqq 23.34 \text{ Dollars}$$

where 4.5 **Dollars** is the *constant term*.

When we carry out the co-multiplication we get the affine inequation

$$3 \cancel{\text{Apples}} \times x \frac{\text{Dollars}}{\cancel{\text{Apple}}} + 4.5 \text{ Dollars} \leqq 23.34 \text{ Dollars}$$

$$[3 \times x] \text{ Dollars} + 4.5 \text{ Dollars} \leqq 23.34 \text{ Dollars}$$

When we factor out the common denominator **Dollars**, we get the *affine* problem in **Dollars**

$$3 \times x + 4.5 \leqq 23.34$$

2. *Translation* problems and *dilation* problems as well as *basic* problems turn out to be special cases of *affine* problems which are therefore a more general type of problems:

- If the number of items in an affine problem is 1, then the affine problem is really just a *translation* problem.

  EXAMPLE 2. If the number of items in EXAMPLE 1 were 1 instead of 3, then the inequation would be

  $$1 \textbf{ Apples} \times x \, \frac{\textbf{Dollars}}{\textbf{Apple}} + 4.5 \textbf{ Dollars} \leqq 23.34 \textbf{ Dollars}$$

  which boils down to the inequation in **Dollars**

  $$x + 4.5 \leqq 23.34$$

  which is a *translation* problem.

- If the *fixed* number-phrase in an affine problem is 0, then that affine problem is really just a *dilation* problem.

  EXAMPLE 3. If the *fixed* number-phrase in EXAMPLE 1 were 0 **Dollars** instead of 4.5 **Dollars**, then the inequation would be

  $$3 \textbf{ Apples} \times x \, \frac{\textbf{Dollars}}{\textbf{Apple}} + 0 \textbf{ Dollars} \leqq 23.35 \textbf{ Dollars}$$

  which boils down to the inequation in **Dollars**

  $$3 \times x \leqq 23.35$$

  which is a *dilation* problem.

- If, in an affine problem, both the additional number-phrase is 0 and the number of items is 1, then that affine problem is really just a *basic* problem.

  EXAMPLE 4. If, in EXAMPLE 24 the number of items were 1 instead of 3 and the additional number-phrase were 0 **Dollars** instead of 4.5 **Dollars**, then the inequation would be

  $$1 \textbf{ Apples} \times x \, \frac{\textbf{Dollars}}{\textbf{Apple}} + 0 \textbf{ Dollars} \leqq 23.35 \textbf{ Dollars}$$

  which boils down to the inequation in **Dollars**

  $$x \leqq 23.35$$

  which is a *basic* problem.

10.2 Solving Affine Problems

We now turn to the investigation of the solution subset of *affine* problems which we will do in accordance with the GENERAL PROCEDURE. The investigation of *affine* problems will proceed much in the same way as that of *translation* and *dilation* problems. As usual, the only difficulty will be that, although similar in nature, problems may involve numerators of different kinds:

- *plain counting* numerators to represent *numbers* of items,
- *signed counting* numerators to represent *two-way numbers* of items,
- *plain decimal* numerators to represent *quantities* of stuff,
- *signed decimal* numerators to represent *two-way quantities* of stuff.

1. We locate the *boundary point* of the solution subset. This involves the following steps:

i. We write the associated equation for the given problem:

EXAMPLE 5. Given the affine problem in **Dollars** in EXAMPLE 1

$$3 \times x + 4.5 \leqq 23.34$$

the associated equation in **Dollars** is

$$3 \times x + 4.5 = 23.34$$

ii. We try to solve the associated equation in *two* stages by way of the REDUCTION APPROACH:

i. We try to reduce the *affine* problem to a *dilation* problem by subtracting the fixed term from *both* sides so as to be able to invoke the **Fairness Theorem**,

ii. We then try to reduce the resulting *dilation* problem to a *basic* problem by dividing by the coefficient of x *both* sides so as to be able to invoke the **Fairness Theorem**.

EXAMPLE 6. Given the affine equation in **Dollars** in EXAMPLE 2

$$3 \times x + 4.5 = 23.34$$

i. We *subtract* 4.5 from *both* sides:

$$3 \times x + 4.5 \; -4.5 = 23.34 \; -4.5$$

which boils down to the dilation equation in **Dollars**

$$3 \times x = 18.84$$

ii. We *divide both* sides by 3

$$3 \times x \; \div 3 = 18.84 \; \div 3$$

which boils down to the *basic* equation in **Dollars**

$$x = 6.28$$

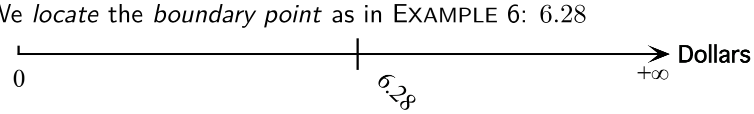
2. We locate the *interior* of the solution subset according to the GENERAL PROCEDURE. (For the sake of completion, we include in the EXAMPLE the step in which we get the *boundary point*.

EXAMPLE 7. Given the *affine* problem in **Dollars** in EXAMPLE 1:

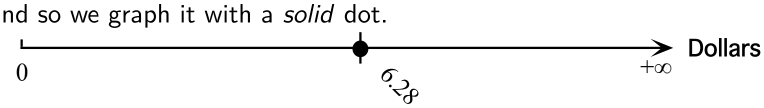
$$3 \times x + 4.5 \leqq 23.34$$

i. To get the *boundary* of the solution subset

i. We *locate* the *boundary point* as in EXAMPLE 6: 6.28

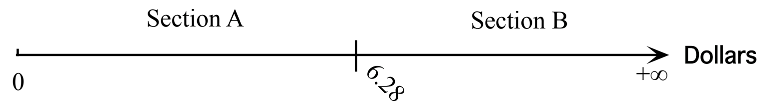


ii. Since the inequation is *lenient*, the *boundary point* is *included* in the solution subset and so we graph it with a *solid* dot.



ii. We locate the *interior* of the solution subset

i. The boundary point 6.28 **Dollars** divides the data set into two sections:



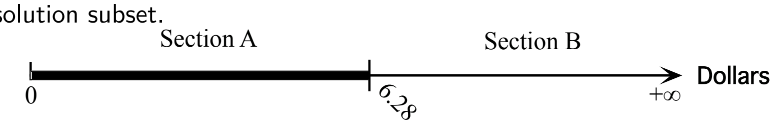
ii. We test Section A with, for instance, 0.1 and, since

$$3 \times x + 4.5 \leqq 23.34|_{x:=0.1} \text{ is TRUE}$$

we get that 0.1 is a *solution* of the inequation in **Dollars**

$$3 \times x + 4.5 \leqq 23.34$$

and **Pasch's Theorem** then tells us that *all* number-phrases in Section A are *included* in the solution subset.



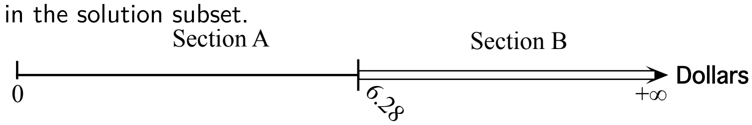
iii. We test Section B with, for instance, $+5.0$ and, since

$$3 \times x + 4.5 \leqq 23.34|_{x:=1000} \text{ is FALSE}$$

we get that 1000 is a *non-solution* of the inequation in **Dollars**

$$3 \times x + 4.5 \leqq 23.34$$

and **Pasch's Theorem** then tells us that *all* number-phrases in Section B are *non-included* in the solution subset.

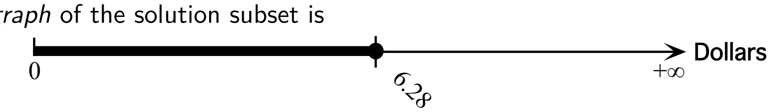


iii. *Altogether*, we represent the solution subset of the inequation in **Dollars**

$$3 \times x + 4.5 \leqq 23.34$$

as follows:

- The *graph* of the solution subset is



- The *name* of the solution subset is

$$(0, 6.28) \text{ Dollars}$$