Chapter 12

Double Affine Problems

We conclude Part Two with double problems which are just like those in the preceding chapter but with *affine* problems instead of *basic* problems.

Conceptually, since *affine* problems can be reduced to *basic* problems, there will be absolutely nothing new in this chapter which serves only to show how much our investment in the PASCH PROCEDURE and the REDUCTION APPROACH will pay.

As a result, the only difficulty will be the “staying power” that will be required by the length of some of the computations.

**EXAMPLE 1.** Solve the double problem in Dollars

\[
\begin{align*}
\text{BOTH} & : +3x + 4.51 \leq +23.35 \\
& +2.34 < +2x
\end{align*}
\]

1. The formula \( +3x + 4.51 \leq +23.35 \) is an *affine* inequation and the formula \( +2.34 < +2x \) is a *basic* inequation so we should be able to find the solution subset on the basis of our previous work. At this point, though, we are not in a position to tell what “named” type of problem this is, if any.

2. We locate the *boundary* of the double problem by looking for the boundary point of each inequation, that is by solving the equation *associated* with each inequation.
   a. The equation associated with the inequation \( 3x + 4.51 \leq +23.35 \) is
      \[
      +3x + 4.51 = +23.35
      \]

i. In order to reduce this *affine* equation to a *basic* equation, we must get rid of \(+4.51\) on the right side which we do by adding its *opposite* \(-4.51\) on both sides so as to be able to invoke the Fairness Theorem:

\[
\begin{align*}
+3x + 4.51 -4.51 &= +23.35 -4.51 \\
+3x &= +18.84
\end{align*}
\]
Then, dividing by $+3$ on both sides

$$+3x \div (+3) = +18.84 \div (+3)$$

gives the basic equation

$$x = +6.28$$

and therefore the boundary point $+6.28$.

**ii.** We check if the boundary point $+6.28$ is included or non-included in the solution subset.

Since we have

$$+3x + 4.51 \leq +23.35\mid_{x=+6.28} \text{ is TRUE}$$

$$+2.34 < +2x\mid_{x=+6.28} \text{ is TRUE}$$

and since, in order for $+6.28$ to be a solution with the connector BOTH, $+6.28$ has to satisfy BOTH formulas, we have that

$$\text{BOTH} \begin{cases} +3x + 4.51 \leq +23.35\mid_{x=+6.28} \text{ is TRUE} \\
+2.34 < +2x\mid_{x=+6.28} \text{ is TRUE} 
\end{cases}$$

so that $+6.28$ is included in the solution subset and we must graph $+6.28$ with a **solid** dot.

**b.** The equation associated with the inequation $+2.34 < +2x$ is:

$$+2.34 = +2x$$

i. We reduce to a basic equation by dividing both sides by $+2$

$$x = +1.17$$

and therefore the boundary point is $+1.17$

**ii.** We check if the boundary point $+1.17$ is included or non-included in the solution subset.

Since we have

$$+3x + 4.51 \leq +23.35\mid_{x=+1.17} \text{ is TRUE}$$

$$+2.34 < +2x\mid_{x=+1.17} \text{ is FALSE}$$

and since, in order for $+1.17$ to be a solution with the connector BOTH, $+1.17$ has to satisfy BOTH formulas, we have that

$$\text{BOTH} \begin{cases} +3x + 4.51 \leq +23.35\mid_{x=+1.17} \text{ is FALSE} \\
+2.34 < +2x\mid_{x=+1.17} \text{ is FALSE} 
\end{cases}$$

so that $+1.17$ is non-included in the solution subset and we must graph $+6.28$ with a **hollow** dot.

**c.** The boundary is

3. We locate the interior of the double problem by testing each one of the three sections determined by the two boundary points:
• We test Section A with, for instance, \(-1000\). That is, we must evaluate the two formulas in the given problem with \(-1000\).

\[
+3x + 4.51 \leq +23.35 | x = -1000 \\
+2.34 < +2x | x = -1000
\]

that is

\[
+3 \cdot (-1000) + 4.51 \leq +23.35 \\
+2.34 < +2 \cdot (-1000)
\]

that is

\[
-3000 + 4.51 \leq +23.35 \\
+2.34 < -2000
\]

that is

\[
-2995.49 \leq +23.35 \quad \text{which is TRUE} \\
+2.34 < -2000 \quad \text{which is FALSE}
\]

Since, in order for \(-1000\) to be a solution with the connector BOTH, \(-1000\) has to satisfy BOTH formulas, we have that

\[
\text{BOTH} \left\{ \begin{align*}
+3x + 4.51 & \leq +23.35 | x = -1000 \\
+2.34 & < +2x | x = -1000
\end{align*} \right. \quad \text{is FALSE}
\]

so that \(-1000\) is non-included in the solution subset. Pasch’s Theorem then tells us that all number-phrases in Section A are non-included in the solution subset.

• We test Section B with, for instance, \(+2\). (We cannot test with 0 since 0 is not in Section B.) That is, we must evaluate the two formulas in the given problem with \(+2\).

\[
+3x + 4.51 \leq +23.35 | x = +2 \\
+2.34 < +2x | x = +2
\]

that is

\[
+3 \cdot (+2) + 4.51 \leq +23.35 \\
+2.34 < +2 \cdot (+2)
\]

that is

\[
+6 + 4.51 \leq +23.35 \\
+2.34 < +4
\]

that is

\[
+10.51 \leq +23.35 \quad \text{which is TRUE} \\
+2.34 < +4 \quad \text{which is TRUE}
\]

Since, in order for \(+2\) to be a solution with the connector BOTH, \(+2\) has to satisfy BOTH formulas, we have that

\[
\text{BOTH} \left\{ \begin{align*}
+3x + 4.51 & \leq +23.35 | x = +2 \\
+2.34 & < +2x | x = +2
\end{align*} \right. \quad \text{is TRUE}
\]

so that \(+2\) is included in the solution subset. Pasch’s Theorem then tells us that all number-phrases in Section B are included in the solution subset.

• We test Section C with, for instance, \(+1000\). That is, we must evaluate the two formulas in the given problem with \(+1000\).
\[ +3x + 4.51 \leq +23.35 \mid x:+1000 \]
\[ +2.34 < +2x \mid x:+1000 \]
that is
\[ +3 \cdot (+1000) + 4.51 \leq +23.35 \]
\[ +2.34 < +2 \cdot (+1000) \]
that is
\[ +3000 + 4.51 \leq +23.35 \]
\[ +2.34 < +2000 \]
that is
\[ +3004.51 \leq +23.35 \quad \text{which is FALSE} \]
\[ +2.34 < +2000 \quad \text{which is TRUE} \]

Since, in order for \(+1000\) to be a solution with the connector BOTH, \(+1000\) has to satisfy BOTH formulas, we have that
\[
\text{BOTH} \left\{ \begin{array}{l}
+3x + 4.51 \leq +23.35 \mid x:+1000 \\
+2.34 < +2x \mid x:+1000
\end{array} \right. \quad \text{is FALSE}
\]
so that \(-1000\) is non-included in the solution subset. Pasch’s Theorem then tells us that all number-phrases in Section A are non-included in the solution subset.

4. We represent and describe the solution subset of the problem in Dollars

\[
\text{BOTH} \left\{ \begin{array}{l}
+3x + 4.51 \leq +23.35 \\
+2.34 < +2x
\end{array} \right.
\]
- The graph of the solution subset is

\[
\begin{array}{c}
\downarrow x > 7.3p \\
\downarrow 7.3p
\end{array}
\]
- The name of the solution subset is
\[ ( +1.17, +6.28 ] \text{ Dollars} \]