Chapter 12

Double Affine Problems

We conclude Part Two with double problems which are just like those in the preceding chapter but with *affine* problems instead of *basic* problems.

Conceptually, since *affine* problems can be reduced to *basic* problems, there will be absolutely nothing new in this chapter which serves only to show how much our investment in thePASCH PROCEDURE and the REDUCTION APPROACH will pay.

As a result, the only difficulty will be the "staying power" that will be required by the length of some of the computations.

EXAMPLE 1. Solve the double problem in **Dollars**

$$\text{BOTH}\begin{cases} +3x + 4.51 \leqq +23.35 \\ +2.34 < +2x \end{cases}$$

1. The formula $+3x + 4.51 \leqq +23.35$ is an *affine* inequation and the formula $+2.34 < +2x$ is a *basic* inequation so we should be able to find the solution subset on the basis of our previous work. At this point, though, we are not in a position to tell what "named" type of problem this is, if any.

2. We locate the *boundary* of the double problem by looking for the boundary point of each inequation, that is by solving the equation *associated* with each inequation.

a. The equation associated with the inequation $3x + 4.51 \leqq +23.35$ is

$$+3x + 4.51 = +23.35$$

i. In order to reduce this *affine* equation to a *basic* equation, we must get rid of $+4.51$ on the right side which we do by adding its *opposite* -4.51 on both sides so as to be able to invoke the FAIRNESS THEOREM:

$$+3x + 4.51 \; -4.51 = +23.35 \; -4.51$$
$$+3x = +18.84$$

Then, dividing by $+3$ on both sides

$$+3x \div(+3) = +18.84 \div(+3)$$

gives the basic equation

$$x = +6.28$$

and therefore the boundary point $+6.28$.
ii. We check if the boundary point $+6.28$ is included or non-included in the solution subset.
Since we have

$$+3x + 4.51 \leqq +23.35|_{x:=+6.28} \text{ is TRUE}$$
$$+2.34 < +2x|_{x:=+6.28} \text{ is TRUE}$$

and since, in order for $+6.28$ to be a solution with the connector BOTH, $+6.28$ has to satisfy BOTH formulas, we have that

$$\text{BOTH} \begin{cases} +3x + 4.51 \leqq +23.35|_{x:=+6.28} \\ +2.34 < +2x|_{x:=+6.28} \end{cases} \text{ is TRUE}$$

so that $+6.28$ is *included* in the solution subset and we must graph $+6.28$ with a *solid* dot.

b. The equation associated with the inequation $+2.34 < +2x$ is:

$$+2.34 = +2x$$

i. We reduce to a *basic* equation by dividing both sides by $+2$

$$x = +1.17$$

and therefore the boundary point is $+1.17$
ii. We check if the boundary point $+1.17$ is included or non-included in the solution subset.
Since we have

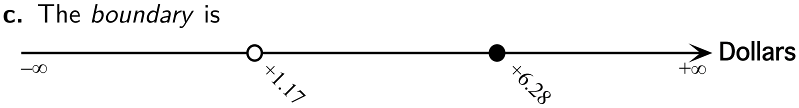
$$+3x + 4.51 \leqq +23.35|_{x:=+1.17} \text{ is TRUE}$$
$$+2.34 < +2x|_{x:=+1.17} \text{ is FALSE}$$

and since, in order for $+1.17$ to be a solution with the connector BOTH, $+1.17$ has to satisfy BOTH formulas, we have that

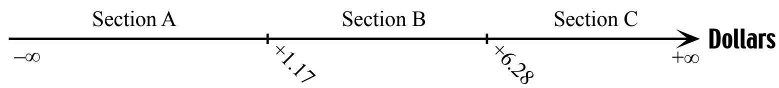
$$\text{BOTH} \begin{cases} +3x + 4.51 \leqq +23.35|_{x:=+1.17} \\ +2.34 < +2x|_{x:=+1.17} \end{cases} \text{ is FALSE}$$

so that $+1.17$ is *non-included* in the solution subset and we must graph $+6.28$ with a *hollow* dot.

c. The *boundary* is



3. We locate the *interior* of the double problem by testing each one of the three sections determined by the two boundary points:



- We test Section A with, for instance, -1000. That is, we must *evaluate* the two formulas in the given problem with -1000.
$$+3x + 4.51 \leqq +23.35|_{x:=-1000}$$
$$+2.34 < +2x|_{x:=-1000}$$
that is
$$+3 \cdot (-1000) + 4.51 \leqq +23.35$$
$$+2.34 < +2 \cdot (-1000)$$
that is
$$-3000 + 4.51 \leqq +23.35$$
$$+2.34 < -2000$$
that is
$$-2995.49 \leqq +23.35 \quad \text{which is TRUE}$$
$$+2.34 < -2000 \quad \text{which is FALSE}$$
Since, in order for -1000 to be a solution with the connector BOTH, -1000 has to satisfy BOTH formulas, we have that
$$\text{BOTH}\begin{cases} +3x + 4.51 \leqq +23.35|_{x:=-1000} \\ +2.34 < +2x|_{x=-1000} \end{cases} \quad \text{is FALSE}$$
so that -1000 is *non-included* in the solution subset. **Pasch's Theorem** then tells us that all number-phrases in Section A are *non-included* in the solution subset.
- We test Section B with, for instance, $+2$. (We cannot test with 0 since 0 is not in Section B.) That is, we must *evaluate* the two formulas in the given problem with $+2$.
$$+3x + 4.51 \leqq +23.35|_{x:=+2}$$
$$+2.34 < +2x|_{x:=+2}$$
that is
$$+3 \cdot (+2) + 4.51 \leqq +23.35$$
$$+2.34 < +2 \cdot (+2)$$
that is
$$+6 + 4.51 \leqq +23.35$$
$$+2.34 < +4$$
that is
$$+10.51 \leqq +23.35 \quad \text{which is TRUE}$$
$$+2.34 < +4 \quad \text{which is TRUE}$$
Since, in order for $+2$ to be a solution with the connector BOTH, $+2$ has to satisfy BOTH formulas, we have that
$$\text{BOTH}\begin{cases} +3x + 4.51 \leqq +23.35|_{x:=+2} \\ +2.34 < +2x|_{x=+2} \end{cases} \quad \text{is TRUE}$$
so that $+2$ is *included* in the solution subset. **Pasch's Theorem** then tells us that all number-phrases in Section B are *included* in the solution subset.
- We test Section C with, for instance, $+1000$. That is, we must *evaluate* the two formulas in the given problem with $+1000$.

$$+3x + 4.51 \leqq +23.35|_{x:=+1000}$$
$$+2.34 < +2x|_{x:=+1000}$$

that is

$$+3 \cdot (+1000) + 4.51 \leqq +23.35$$
$$+2.34 < +2 \cdot (+1000)$$

that is

$$+3000 + 4.51 \leqq +23.35$$
$$+2.34 < +2000$$

that is

$$+3004.51 \leqq +23.35 \quad \text{which is FALSE}$$
$$+2.34 < +2000 \quad \text{which is TRUE}$$

Since, in order for $+1000$ to be a solution with the connector BOTH, $+1000$ has to satisfy BOTH formulas, we have that

$$\text{BOTH} \begin{cases} +3x + 4.51 \leqq +23.35|_{x:=+1000} \\ +2.34 < +2x|_{x=+1000} \end{cases} \quad \text{is FALSE}$$

so that -1000 is *non-included* in the solution subset. **Pasch's Theorem** then tells us that all number-phrases in Section A are *non-included* in the solution subset.

4. We represent and describe the solution subset of the problem in **Dollars**

$$\text{BOTH} \begin{cases} +3x + 4.51 \leqq +23.35 \\ +2.34 < +2x \end{cases}$$

- The *graph* of the solution subset is

- The *name* of the solution subset is

$$\big(+1.17, +6.28 \big] \text{ Dollars}$$