split equality
common base

Chapter 14

Laurent Monomials

Because of the lack of a *short* procedure for identifying monomial specifying-phrases, when working with monomial specifying-phrases, we tend to delay identifying them as much as possible and, instead, to *compute* with the monomial specifying-phrases themselves as long as possible, that is until there is nothing else to do but to *identify* the resulting monomial specifying-phrase.

NOTE. The format that we will use to write these computations is called **split equality**: We will write on the left the (compound) specifying-phrase that we want to identify and we will write on the right the successive stages of the *computation* on separate lines.

14.1 Multiplying Monomial Specifying-Phrases

When we *multiply* two monomial specifying-phrases with a **common base**, that is when we multiply a first monomial specifying-phrase by a second monomial specifying-phrase with the same base, the result turns out to be a monomial specifying-phrase with the *common base*[1].

1. We can get this result either one of two ways:

- We can go back to the *in-line templates*:

  i. We replace each *monomial specifying-phrase* by the corresponding *in-line* template,

  ii. We change the order of the multiplications,

  iii. We write the resulting monomial specifying phrase.

[1]Educologists will have recognized multiplication as a binary operation.

EXAMPLE 1. In order to identify

$$\left[17 \times 2^{+5}\right] \times \left[11 \times 2^{+4}\right]$$

we replace each *monomial specifying-phrase* by the corresponding *in-line* template, we change the order of the multiplications and we write the resulting monomial specifying-phrase:

$$\begin{aligned}\left[17 \times 2^{+5}\right] \times \left[11 \times 2^{+4}\right] &= \left[17 \times 2 \times 2 \times 2 \times 2 \times 2\right] \times \left[11 \times 2 \times 2 \times 2 \times 2\right] \\ &= 17 \times 2 \times 2 \times 2 \times 2 \times 2 \ \times\ 11 \times 2 \times 2 \times 2 \times 2 \\ &= 17 \times 11 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= \left[17 \times 11\right] \times 2^{+(5+4)} \\ &= 187 \times 2^{+12}\end{aligned}$$

- We can build the resulting monomial specifying-phrase right from the given monomial specifying-phrases using the following procedure:
  i. We get the *coefficient* of the resulting monomial specifying-phrase by multiplying the coefficients of the given monomial specifying-phrases,
  ii. We get the *base* of the resulting monomial specifying-phrase by taking the base common to the given monomial specifying-phrases,
  iii. We get the signed exponent of the resulting monomial specifying-phrase by "oplussing" the signed exponents of the given monomial specifying-phrases.

EXAMPLE 2. In order to identify

$$\left[17 \times 2^{+5}\right] \times \left[11 \times 2^{+4}\right]$$

we multiply the coefficients and we "oplus" the signed exponents:

$$\begin{aligned}\left[17 \times 2^{+5}\right] \times \left[11 \times 2^{+4}\right] &= \left[17 \times 11\right] \times 2^{+5 \oplus +4} \\ &= 187 \times 2^{+12}\end{aligned}$$

2. In order to see why both ways give the same result, we now look at three more examples in which we will get the result both ways[2].

EXAMPLE 3. We identify

$$\left[17 \times 2^{+5}\right] \times \left[11 \times 2^{-2}\right]$$

both ways:

- We replace each *monomial specifying-phrase* by the corresponding *in-line* template, change the order of the multiplications and write the resulting monomial specifying-phrase:

$$\left[17 \times 2^{+5}\right] \times \left[11 \times 2^{-2}\right] = \left[17 \times 2 \times 2 \times 2 \times 2 \times 2\right] \times \left[\frac{11}{2 \times 2}\right]$$

[2] Educologists will of course approve of letting the students "experience" the amount of work being saved by having them do it both ways for a while.

$$
\begin{aligned}
&= \frac{17 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11}{2 \times 2} \\
&= \frac{17 \times 11 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2} \\
&= \frac{17 \times 11 \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times 2}{\cancel{2} \times \cancel{2}} \\
&= 17 \times 11 \times 2 \times 2 \times 2 \\
&= 17 \times 11\big] \times 2^{+(5-2)} \\
&= 187 \times 2^{+3}
\end{aligned}
$$

- We multiply the coefficients and we "oplus" the signed exponents:

$$
\begin{aligned}
\big[17 \times 2^{+5}\big] \times \big[11 \times 2^{-2}\big] &= \big[17 \times 11\big] \times 2^{+5 \oplus -2} \\
&= 187 \times 2^{+3}
\end{aligned}
$$

EXAMPLE 4. We identify

$$\big[17 \times 2^{-6}\big] \times \big[11 \times 2^{+2}\big]$$

both ways:

- We replace each *monomial specifying-phrase* by the corresponding *in-line* template, change the order of the multiplications and write the resulting monomial specifying-phrase:

$$
\begin{aligned}
\big[17 \times 2^{-6}\big] \times \big[11 \times 2^{+2}\big] &= \left[\frac{17}{2 \times 2 \times 2 \times 2 \times 2 \times 2}\right] \times \big[11 \times 2 \times 2\big] \\
&= \frac{17 \times 11 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2} \\
&= \frac{17 \times 11 \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times 2 \times 2 \times 2 \times 2} \\
&= \frac{17 \times 11}{2 \times 2 \times 2 \times 2} \\
&= \big[17 \times 11\big] \times 2^{-(6-2)} \\
&= 187 \times 2^{-4}
\end{aligned}
$$

- We multiply the coefficients and we "oplus" the signed exponents:

$$
\begin{aligned}
\big[17 \times 2^{-6}\big] \times \big[11 \times 2^{+2}\big] &= \big[17 \times 11\big] \times 2^{-6 \oplus +2} \\
&= 187 \times 2^{-4}
\end{aligned}
$$

EXAMPLE 5. We identify

common base

$$\left[17 \times 2^{-4}\right] \times \left[11 \times 2^{-3}\right]$$

both ways:

- We replace each *monomial specifying-phrase* by the corresponding *in-line* template, change the order of the multiplications and write the resulting monomial specifying-phrase:

$$\begin{aligned}\left[17 \times 2^{-4}\right] \times \left[11 \times 2^{-3}\right] &= \left[\frac{17}{2 \times 2 \times 2 \times 2}\right] \times \left[\frac{11}{2 \times 2 \times 2}\right] \\ &= \frac{17 \times 11}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} \\ &= \left[17 \times 11\right] \times 2^{-(4+3)} \\ &= 187 \times 2^{-7}\end{aligned}$$

- We multiply the coefficients and we "oplus" the signed exponents:

$$\begin{aligned}\left[17 \times 2^{-4}\right] \times \left[11 \times 2^{-3}\right] &= \left[17 \times 11\right] \times 2^{-4 \oplus -3} \\ &= 187 \times 2^{-7}\end{aligned}$$

3. Thus, from the above examples, we see that the "power language" is indeed powerful as it allows for a single procedure since the "oplus" automatically takes care of the different cases whereas, when we use in-line templates, we need different procedures depending on whether the coefficients are to be repeatedly multiplied or divided by the copies of the base and also on the relative number of copies when one coefficient is to be repeatedly multiplied while the other coefficient is to be repeatedly divided.

14.2 Dividing Monomial Specifying-Phrases

When we divide two monomial specifying-phrases with a **common base**, that is when we divide a first monomial specifying-phrase by a second monomial specifying-phrase with the same base, the result turns out to be a monomial specifying-phrase with the same base.

1. We can get the result either one of two ways:

- We can go back to the *in-line templates*:

  i. We replace each *monomial specifying-phrase* by the corresponding *in-line* template, using fraction bars,

  ii. We "invert and multiply", change the order of the multiplications, cancel, etc

  iii. We write the resulting monomial specifying phrase.

EXAMPLE 6. In order to identify

$$\left[17 \times 2^{+7}\right] \div \left[11 \times 2^{+3}\right]$$

We replace each *monomial specifying-phrase* by the corresponding *in-line* template using fraction bars, "invert and multiply", change the order of the multiplications, cancel and write the resulting monomial specifying-phrase:

$$\begin{aligned}
\left[17 \times 2^{+7}\right] \div \left[11 \times 2^{+3}\right] &= \frac{17 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{1} \div \frac{11 \times 2 \times 2 \times 2}{1} \\
&= \frac{17 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{1} \times \frac{1}{11 \times 2 \times 2 \times 2} \\
&= \frac{17 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{11 \times 2 \times 2 \times 2} \\
&= \frac{17}{11} \times \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} \\
&= \frac{17}{11} \times \frac{\not{2} \times \not{2} \times \not{2} \times 2 \times 2 \times 2 \times 2}{\not{2} \times \not{2} \times \not{2}} \\
&= \frac{17}{11} \times \frac{2 \times 2 \times 2 \times 2}{1} \\
&= \frac{17}{11} \times 2 \times 2 \times 2 \times 2 \\
&= \frac{17}{11} \times 2^{+(7-3)} \\
&= \frac{17}{11} \times 2^{+4}
\end{aligned}$$

- We can build the resulting monomial specifying-phrase right from the given monomial specifying-phrases:

  i. We get the *coefficient* of the resulting monomial specifying-phrase by dividing the coefficients of the given monomial specifying-phrases,

  ii. We get the *base* of the resulting monomial specifying-phrase by taking the base common to the given monomial specifying-phrases,

  iii. We get the signed exponent of the resulting monomial specifying-phrase by "ominussing" the signed exponent of the second given monomial specifying-phrase from the signed exponent of the first given monomial specifying-phrase, that is by "oplussing" the *opposite* of the signed exponent of the second given monomial specifying-phrase to the signed exponent of the first given monomial specifying-phrase.

EXAMPLE 7. In order to identify

$$\left[17 \times 2^{+7}\right] \div \left[11 \times 2^{+3}\right]$$

We divide the coefficients and we "ominus" the signed exponents:

$$\left[17 \times 2^{+7}\right] \div \left[11 \times 2^{+3}\right] = \left[17 \div 11\right] \times 2^{+7 \ominus +3}$$

$$= \frac{17}{11} \times 2^{+7 \oplus -3}$$
$$= \frac{17}{11} \times 2^{+4}$$

2. In order to see why both ways give the same result, we now look at three more examples the result of each of which we will get both ways.

EXAMPLE 8. We identify

$$\left[17 \times 2^{+7}\right] \div \left[11 \times 2^{+3}\right]$$

both ways:

- We replace each *monomial specifying-phrase* by the corresponding *in-line* template using fraction bars, "invert and multiply", change the order of the multiplications, cancel and write the resulting monomial specifying-phrase:

$$\begin{aligned}
\left[17 \times 2^{+7}\right] \div \left[11 \times 2^{+3}\right] &= \frac{17 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{1} \div \frac{11 \times 2 \times 2 \times 2}{1} \\
&= \frac{17 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{1} \times \frac{1}{11 \times 2 \times 2 \times 2} \\
&= \frac{17 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{11 \times 2 \times 2 \times 2} \\
&= \frac{17}{11} \times \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} \\
&= \frac{17}{11} \times \frac{\not{2} \times \not{2} \times \not{2} \times 2 \times 2 \times 2 \times 2}{\not{2} \times \not{2} \times \not{2}} \\
&= \frac{17}{11} \times \frac{2 \times 2 \times 2 \times 2}{1} \\
&= \frac{17}{11} \times 2 \times 2 \times 2 \times 2 \\
&= \frac{17}{11} \times 2^{+(7-3)} \\
&= \frac{17}{11} \times 2^{+4}
\end{aligned}$$

- We divide the coefficients and we "ominus" the signed exponents:

$$\begin{aligned}
\left[17 \times 2^{+7}\right] \div \left[11 \times 2^{+3}\right] &= \left[17 \div 11\right] \times 2^{+7 \ominus +3} \\
&= \frac{17}{11} \times 2^{+7 \oplus -3} \\
&= \frac{17}{11} \times 2^{+4}
\end{aligned}$$

EXAMPLE 9. We identify

$$\left[17 \times 2^{+3}\right] \div \left[11 \times 2^{+7}\right]$$

both ways:

- We replace each *monomial specifying-phrase* by the corresponding *in-line* template using a fraction bar, change the order of the multiplications, cancel and write the resulting monomial specifying-phrase:

$$\begin{aligned}
\left[17 \times 2^{+7}\right] \div \left[11 \times 2^{+3}\right] &= \frac{17 \times 2 \times 2 \times 2}{1} \div \frac{11 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{1} \\
&= \frac{17 \times 2 \times 2 \times 2}{1} \div \frac{1}{11 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} \\
&= \frac{17 \times 2 \times 2 \times 2}{11 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} \\
&= \frac{17}{11} \times \frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} \\
&= \frac{17}{11} \times \frac{\cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times 2 \times 2} \\
&= \frac{17}{11} \times \frac{1}{2 \times 2 \times 2 \times 2} \\
&= \frac{17}{11} \times 2^{-(7-3)} \\
&= \frac{17}{11} \times 2^{-4}
\end{aligned}$$

- We divide the coefficients and we "ominus" the signed exponents:

$$\begin{aligned}
\left[17 \times 2^{+3}\right] \div \left[11 \times 2^{+7}\right] &= \left[17 \div 11\right] \times 2^{+3 \ominus +7} \\
&= \frac{17}{11} \times 2^{+3 \oplus -7} \\
&= \frac{17}{11} \times 2^{-4}
\end{aligned}$$

EXAMPLE 10. We identify

$$\left[17 \times 2^{-5}\right] \div \left[11 \times 2^{+3}\right]$$

both ways:

- We replace each *monomial specifying-phrase* by the corresponding *in-line* template using a fraction bar, change the order of the multiplications, cancel and write the resulting monomial specifying-phrase:

$$\begin{aligned}
\left[17 \times 2^{-5}\right] \div \left[11 \times 2^{+3}\right] &= \frac{17}{2 \times 2 \times 2 \times 2 \times 2} \div \frac{11 \times 2 \times 2 \times 2}{1} \\
&= \frac{17}{2 \times 2 \times 2 \times 2 \times 2} \times \frac{1}{11 \times 2 \times 2 \times 2} \\
&= \frac{17 \times 1}{11 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{17}{11} \times \frac{1}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} \\
&= \frac{17}{11} \times 2^{-(5+3)} \\
&= \frac{17}{11} \times 2^{-8}
\end{aligned}
$$

- We divide the coefficients and we "ominus" the signed exponents:

$$
\begin{aligned}
\left[17 \times 2^{-5}\right] \div \left[11 \times 2^{+3}\right] &= \left[17 \div 11\right] \times 2^{-5 \ominus +3} \\
&= \frac{17}{11} \times 2^{-5 \oplus -3} \\
&= \frac{17}{11} \times 2^{-8}
\end{aligned}
$$

3. Thus, from the above examples, we see that the "power language" is even more spectacular in the case of division as the "ominus" still takes automatically care of the different cases while, whereas, we use in-line templates, we need different procedures depending on whether the coefficients are to be repeatedly multiplied or divided by the copies of the base and also on the relative number of copies when one coefficient is to be repeatedly multiplied while the other coefficient is to be repeatedly divided.

4. The reason we are using *Laurent* monomial specifying-phrases rather than just *plain* monomial specifying-phrases is that we cannot always divide a first *plain* monomial specifying-phrase by a second *plain* monomial specifying-phrase and get as a result a *plain* monomial specifying-phrase. On the other hand, we can always multiply or divide a first *Laurent* monomial specifying-phrase by a second *Laurent* monomial specifying-phrase and get as a result a *Laurent* monomial specifying-phrase.

14.3 Terms

We now take a major step in the development of the "power language" by allowing *unspecified numerators* when writing monomials.

1. We begin by going back to the distinction between a *formula* and a *sentence*. Recall that by itself a *formula*, for instance an *inequation* or an *equation*, is neither TRUE nor FALSE and that only a *sentence* can represent a relationship among collections in the real-world.

EXAMPLE 11. The inequation in **Apples**

$$x < 5$$

is neither TRUE nor FALSE because it does not represent a relationship among collections in the real world. (2 **Apples** represent a collection in the real world but x **Apples** does not represent a collection in the real world.) term

Given a formula, it is only when we replace the *unspecified numerator* by a *specific numerator* that we get a *sentence* which is then either TRUE or FALSE depending on whether it fits the real world or not.

EXAMPLE 12. Given the *formula* in **Apples**

$$x < 5$$

when we replace the unspecified numerator x by the specific numerator 8 we get the sentence in **Apples**

$$x{<}5|_{x:=8}$$

that is the sentence

$$8 \text{ Apples} < 5 \text{ Apples}$$

which is FALSE but if, instead, we replace the unspecified numerator x by the specific numerator 3 we get the sentence in **Apples**

$$x{<}5|_{x:=3}$$

that is the sentence

$$3 \text{ Apples} < 5 \text{ Apples}$$

which is TRUE

2. Similarly, just as a *formula* can be viewed as an "incomplete" *sentence*, a **term** will be an "incomplete" *specifying-phrase*.

EXAMPLE 13. Given the *term* in **Apples**

$$x + 5$$

when we replace the unspecified numerator x by the specific numerator 8 we get the specifying-phrase in **Apples**

$$x + 5|_{x:=8}$$

that is the specifying-phrase

$$8 \text{ Apples} + 5 \text{ Apples}$$

which we may or may not chose to *identify*.

Of course, an *unspecified numerator* is the simplest possible kind of *term*.

EXAMPLE 14. Given the *term* in **Apples**

$$x$$

when we replace the unspecified numerator x by the specific numerator 8

$$x + 5|_{x:=8}$$

we get

$$8 \text{ Apples}$$

3. When replacing in a *monomial* specifying-phrase a specific numerator by an unspecified numerator to get a *term*, we will use

- The letters $a, b, c, d \dots$ for unspecified *signed* coefficients,
- The letters $x, y, x \dots$ for unspecified *signed* bases,
- The letters $m, n, p \dots$ for unspecified *plain* exponents.

EXAMPLE 15.

monomial term
monomial

$$a \times x^{+n}$$
$$c \times y^{-m}$$

The reason we will use the letters $m, n, p \dots$ to stand only for *plain* exponents (rather than for *signed* exponents) is that the *sign* of a exponent is most important since it distinguishes between multiplication and division and we will almost always have to specify it as in the above example.

In the rare cases when the sign of the exponent will not matter, we will write the symbol $\pm$, read "plus or minus" in front of the letter as in the following example.

Example 16.

$$c \times x^{\pm n}$$

is intended to cover both the case

$$c \times x^{+n}$$

and the case

$$c \times x^{-n}$$

It is also customary to let the separator $\times$ go without saying. However, this tends to cause mistakes unless we make sure we read the monomial specifying-phrase according to whether the signed exponent is *positive* or *negative*, as

- "Coefficient *multiplied* by number of copies of the base" when the exponent is *positive*,
- "Coefficient *divided* by number of copies of the base" when the exponent is *negative*.

Example 17.

- We read cx^{+n} as "c multiplied by n copies of x" because the exponent is *positive*,
- We read ay^{-p} as "a divided by p copies of y" because the exponent is *negative*.

14.4 Monomials

In the rest of this text, coefficients and exponents will always be specified and only the base will remain unspecified. Out of habit, we shall mostly use the letter x for the base.

1. Monomial specifying-phrases in which the *base* is unspecified are called **monomial terms** or **monomials** for short.

Example 18. The following

$$-3x^{+5}$$
$$+5.23x^{-3}$$
$$-1600x^{-4}$$
$$+4x^{+2}$$

are monomials but

$$+4x^{+2.5}$$

is not a monomial because 2.5 copies doesn't make sense.

Laurent monomial
plain monomial
coefficient
power

a. Just as, earlier on, we distinguished *Laurent* monomial specifying-phrases (those whose exponent can have any sign) from *plain* monomial specifying phrases (those whose exponent can be only positive or 0), we could distinguish in the same manner **Laurent monomials** from **plain monomials**. However, since we will be using mostly *Laurent* monomials, we will just use *monomial* to mean Laurent monomial.

b. In a *monomial* we will distinguish:

- the **coefficient**, which is the number to be multiplied or divided by the copies of the base
- the **power**, which is the base together with the exponent.

In other words, the *separator* $\times$, whether it is actually written or goes without saying, separates the *coefficient* from the *power*.

EXAMPLE 19. In the monomial $-3x^{+4}$, -3 is the *coefficient* and x^{+4} is the *power*.

c. Thus, *monomials*, as well as *monomial specifying-phrases*, look very much like *ordinary* number-phrases (as opposed to *specifying* number-phrases):

- The *coefficient* in a monomial—or monomial specifying-phrase—is like the *numerator* in an ordinary number-phrase,
- The *power* in a monomial—or monomial specifying-phrase—is like the *denominator* in an ordinary number-phrase.

EXAMPLE 20. Monomial specifying-phrases like

$17.52 \times 2^{+3}$ (with $\times$ as *separator*)

and monomials like

$17.52\, x^{+3}$ (without *separator*)

look, and to a large extent will behave, very much like:

- Ordinary number-phrases like

  17.52 **Meters**

  in which there is no need for a *separator* between the *numerator* and the *denominator*,
- Metric number-phrases like

  17.52 **Kilo Meters**

  in which there is no need for a *separator* between the *numerator* and the *denominator*,
- Base TEN number-phrases like

  17.52 **x Ten^{+3} Meters**

  where $\times$ is a *separator* between the *numerator* and the *denominator*,
- Exponential number-phrases like

  17.52 **x 10^{+3} Meters**

  where $\times$ is a *separator* between the *numerator* and the *denominator*.

We will investigate how far the similarity goes in the following chapters.

2. When we multiply or divide a first monomial by a second monomial, we proceed just as we did with monomial specifying-phrases, that is we can proceed either:

- The long way which is to go back to in-line templates and then proceed according to whether we are dealing with *multiplication* or *division*
- The short way which is to use the following

THEOREM 8 (EXPONENT THEOREM). *In order to:*
i. Multiply *two monomials* $ax^{\pm m}$ *and* $bx^{\pm n}$, *we* multiply the coefficients *and* oplus the exponents*:*

$$ax^{\pm m} \times bx^{\pm n} = abx^{\pm m \oplus \pm n}$$

ii. Divide *two monomials* $ax^{\pm m}$ *and* $bx^{\pm n}$, *we* divide the coefficients *and* ominus the exponents*:*

$$ax^{\pm m} \div bx^{\pm n} = \frac{a}{b}x^{\pm m \ominus \pm n}$$

We now look at a few examples.

EXAMPLE 21. Given

$$\left[-17.89 \times x^{+547}\right] \times \left[-11.06 \times x^{+312}\right]$$

instead of replacing each *monomial* by the corresponding *in-line* template, change the order of the multiplications and write the resulting monomial:

$$\begin{aligned}\left[-17.89 \times x^{+547}\right] \times \left[-11.06 \times x^{+312}\right] &= \left[-17.89 \times \underbrace{x \times x \times \cdots \times x}_{\text{547 copies of } x}\right] \times \left[-11.06 \times \underbrace{x \times x \times \cdots \times x}_{\text{312 copies of } x}\right] \\ &= -17.89 \times -11.06 \times \underbrace{x \times x \times \cdots \times x}_{\text{547+312 copies of } x} \\ &= \left[-17.89 \times -11.06\right] \times x^{+(547+312)} \\ &= +\left[17.89 \times 11.06\right] \times x^{+859}\end{aligned}$$

we can use the EXPONENT THEOREM:

$$\begin{aligned}\left[-17.89 \times x^{+547}\right] \times \left[-11.06 \times x^{+312}\right] &= \left[-17.89 \times -11.06\right] \times x^{+547 \oplus +312} \\ &= +\left[17.89 \times 11.06\right] \times x^{+859}\end{aligned}$$

EXAMPLE 22. Given

$$\left[+17.89 \times x^{+547}\right] \times \left[-11.06 \times x^{-312}\right]$$

instead of replacing each *monomial* by the corresponding *in-line* template, change the order of the multiplications and write the resulting monomial:

$$\begin{aligned}
\left[+17.89 \times x^{+547}\right] \times \left[-11.06 \times x^{-312}\right] &= \left[+17.89 \times \underbrace{x \times x \times \cdots \times x}_{547 \text{ copies of } x}\right] \times \left[\frac{-11.06}{\underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x}}\right] \\
&= \left[+17.89 \times -11.06\right] \times \left[\frac{\underbrace{x \times x \times \cdots \times x}_{547 \text{ copies of } x}}{\underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x}}\right] \\
&= -\left[17.89 \times 11.06\right] \times \left[\frac{\underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x} \times \underbrace{x \times x \times \cdots \times x}_{547-312 \text{ copies of } x}}{\underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x}}\right] \\
&= -\left[17.89 \times 11.06\right] \times x^{+(547-312)} \\
&= -\left[17.89 \times 11.06\right] \times x^{+235}
\end{aligned}$$

we can use the EXPONENT THEOREM:

$$\begin{aligned}
\left[+17.89 \times x^{+547}\right] \times \left[-11.06 \times x^{-312}\right] &= \left[+17.89 \times -11.06\right] \times x^{+547 \oplus -312} \\
&= -\left[17.89 \times 11.06\right] \times x^{+(547-312)} \\
&= -\left[17.89 \times 11.06\right] \times x^{+235}
\end{aligned}$$

EXAMPLE 23. Given

$$\left[-17.89 \times x^{-547}\right] \times \left[+11.06 \times x^{+312}\right]$$

instead of replacing each *monomial* by the corresponding *in-line* template, change the order of the multiplications and write the resulting monomial:

$$\begin{aligned}
\left[-17.89 \times x^{-547}\right] \times \left[+11.06 \times x^{+312}\right] &= \left[\frac{-17.89}{\underbrace{x \times x \times \cdots \times x}_{547 \text{ copies of } x}}\right] \times \left[+11.06 \times \underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x}\right] \\
&= \left[-17.89 \times +11.06\right] \times \left[\frac{\underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x}}{\underbrace{x \times x \times \cdots \times x}_{547 \text{ copies of } x}}\right]
\end{aligned}$$

$$= -\Big[17.89 \times 11.06\Big] \times \left[\frac{\underbrace{\cancel{x \times x \times \cdots \times x}}_{\cancel{312 \text{ copies of } x}}}{\underbrace{\cancel{x \times x \times \cdots \times x}}_{\cancel{312 \text{ copies of } x}} \times \underbrace{x \times x \times \cdots \times x}_{547-312 \text{ copies of } x}}\right]$$

$$= -\Big[17.89 \times 11.06\Big] \times x^{-(547-312)}$$

$$= -\Big[17.89 \times 11.06\Big] \times x^{-235}$$

we can use the EXPONENT THEOREM:

$$\Big[-17.89 \times x^{-547}\Big] \times \Big[11.06 \times x^{+312}\Big] = \Big[-17.89 \times +11.06\Big] \times x^{-547 \oplus +312}$$

$$= -\Big[17.89 \times 11.06\Big] \times x^{-(547-312)}$$

$$= -\Big[17.89 \times 11.06\Big] \times x^{-235}$$

EXAMPLE 24. Given

$$\Big[+17.89 \times x^{+547}\Big] \div \Big[+11.06 \times x^{+312}\Big]$$

instead of replacing each *monomial* by the corresponding *in-line* template, change the order of the multiplications, rewrite as fraction, multiply by the reciprocal instead of divide, and write the resulting monomial:

$$\Big[+17.89 \times x^{+547}\Big] \div \Big[+11.06 \times x^{+312}\Big] = \left[+17.89 \times \underbrace{x \times x \times \cdots \times x}_{547 \text{ copies of } x}\right] \div \left[\frac{+11.06 \times \underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x}}{1}\right]$$

$$= \left[+17.89 \times \underbrace{x \times x \times \cdots \times x}_{547 \text{ copies of } x}\right] \times \left[\frac{1}{+11.06 \times \underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x}}\right]$$

$$= \left[\frac{+17.89}{+11.06}\right] \times \left[\frac{\underbrace{x \times x \times \cdots \times x}_{547 \text{ copies of } x}}{\underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x}}\right]$$

$$= +\left[\frac{17.89}{11.06}\right] \times \left[\frac{\underbrace{\cancel{x \times x \times \cdots \times x}}_{\cancel{312 \text{ copies of } x}} \times \underbrace{x \times x \times \cdots \times x}_{547-312 \text{ copies of } x}}{\underbrace{\cancel{x \times x \times \cdots \times x}}_{\cancel{312 \text{ copies of } x}}}\right]$$

$$= +\left[\frac{17.89}{11.06}\right] \times x^{+(547-312)}$$

$$= + \left[\frac{17.89}{11.06}\right] \times x^{+235}$$

it is easier to use the EXPONENT THEOREM:

$$\left[+17.89 \times x^{+547}\right] \div \left[+11.06 \times x^{+312}\right] = \left[\frac{+17.89}{+11.06}\right] \times x^{+547 \ominus +312}$$
$$= + \left[\frac{17.89}{11.06}\right] \times x^{+547 \oplus -312}$$
$$= + \left[\frac{17.89}{11.06}\right] \times x^{+(547-312)}$$
$$= + \left[\frac{17.89}{11.06}\right] \times x^{+235}$$

Example 25. Given

$$\left[17.89 \times x^{-547}\right] \div \left[11.06 \times x^{-312}\right]$$

instead of replacing each *monomial* by the corresponding *in-line* template, change the order of the multiplications, rewrite as fraction, multiply by the reciprocal instead of divide, and write the resulting monomial:

$$\left[17.89 \times x^{-547}\right] \div \left[11.06 \times x^{-312}\right] = \left[\frac{17.89}{\underbrace{x \times x \times \cdots \times x}_{547 \text{ copies of } x}}\right] \div \left[\frac{11.06}{\underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x}}\right]$$
$$= \left[\frac{17.89}{\underbrace{x \times x \times \cdots \times x}_{547 \text{ copies of } x}}\right] \times \left[\frac{\underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x}}{11.06}\right]$$
$$= \left[\frac{17.89}{11.06}\right] \times \left[\frac{\underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x}}{\underbrace{x \times x \times \cdots \times x}_{547 \text{ copies of } x}}\right]$$
$$= \left[\frac{17.89}{11.06}\right] \times \left[\frac{\cancel{\underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x}}}{\cancel{\underbrace{x \times x \times \cdots \times x}_{312 \text{ copies of } x}} \times \underbrace{x \times x \times \cdots \times x}_{547-312 \text{ copies of } x}}\right]$$
$$= \left[\frac{17.89}{11.06}\right] \times x^{-(547-312)}$$

$$= \left[\frac{17.89}{11.06}\right] \times x^{-235}$$

it is easier to use the EXPONENT THEOREM:

$$\left[17.89 \times x^{-547}\right] \div \left[11.06 \times x^{-312}\right] = \left[\frac{17.89}{11.06}\right] \times x^{-547 \ominus -312}$$
$$= \left[\frac{17.89}{11.06}\right] \times x^{-547 \oplus +312}$$
$$= \left[\frac{17.89}{11.06}\right] \times x^{-(547-312)}$$
$$= \left[\frac{17.89}{11.06}\right] \times x^{-235}$$