MATH 017 REVIEW III Discussions

[Run: 04/02/2013 at 21:43 Seed: 7124. Order of Checkable Items: List.]

III-1. Identify the monomial specifying-phrase in **Dollars**: $3 \times 5^{+4}$

Discussion: We go through the following steps:

- We *read* $3 \times 5^{+4}$ as 3 *multiplied* by 4 copies of 5
- We *write*: $3 \times 5^{+4} = 3 \times 5 \times 5 \times 5 \times 5$
- We *compute*: $= 1875$

III-2. Identify the monomial specifying-phrase in **Dollars**: 256×2^{-5}

Discussion: We go through the following steps:

- We *read* 256×2^{-5} as 256 *divided* by 5 copies of 2
- We *write*: $256 \times 2^{-5} = \dfrac{256}{2 \times 2 \times 2 \times 2 \times 2}$
- We *compute*: $= 8$

III-3. Identify the monomial specifying-phrase in **Dollars**: $-162 \times (-3)^{-2}$

Discussion:

- We *read* $-162 \times (-3)^{-2}$ as -162 *divided* by 2 copies of -3
- We *write*: $-162 \times (-3)^{-2} = \dfrac{-162}{(-3) \times (-3)}$
- We *compute*: $= -18$

III-4. Identify $\left[-12x^{+5}\right] \otimes \left[+4x^{+3}\right]$

Discussion: In order to *multiply* monomial specifying-phrase with a common basis,

i. We get the *coefficient* of the resulting monomial specifying-phrase by *multiplying* the coefficients of the given monomial specifying-phrases:

$$-12 \otimes +4$$

ii. We get the *base* of the resulting monomial specifying-phrase by taking the base common to the given monomial specifying-phrases:

$$x$$

iii. We get the signed exponent of the resulting monomial specifying-phrase by "oplussing" the signed exponents of the given monomial specifying-phrases:

$$+5 \oplus +3$$

So, we write and compute:

$$\begin{aligned}\left[-12x^{+5}\right] \otimes \left[+4x^{+3}\right] &= [-12 \otimes +4]\, x^{+5 \,\oplus\, +3} \\ &= -48x^{+8}\end{aligned}$$

(Since x is unspecified we cannot go any further.)

III-5. Identify $[-12x^{+5}] \otimes [-4x^{-3}]$

Discussion: In order to *multiply* monomial specifying-phrase with a common basis,

i. We get the *coefficient* of the resulting monomial specifying-phrase by *multiplying* the coefficients of the given monomial specifying-phrases:

$$-12 \otimes -4$$

ii. We get the *base* of the resulting monomial specifying-phrase by taking the base common to the given monomial specifying-phrases:

$$x$$

iii. We get the signed exponent of the resulting monomial specifying-phrase by "oplussing" the signed exponents of the given monomial specifying-phrases:

$$+5 \oplus -3$$

So, we write and compute:

$$\begin{aligned}\left[-12x^{+5}\right] \otimes \left[-4x^{-3}\right] &= [-12 \otimes -4]\, x^{+5 \,\oplus\, -3} \\ &= -48x^{+2}\end{aligned}$$

(Since x is unspecified we cannot go any further.)

III-6. Identify $[+12x^{-5}] \otimes [+4x^{+3}]$

Discussion: In order to *multiply* monomial specifying-phrase with a common basis,

i. We get the *coefficient* of the resulting monomial specifying-phrase by *multiplying* the coefficients of the given monomial specifying-phrases:

$$+12 \otimes +4$$

ii. We get the *base* of the resulting monomial specifying-phrase by taking the base common to the given monomial specifying-phrases:

$$x$$

iii. We get the signed exponent of the resulting monomial specifying-phrase by "oplussing" the signed exponents of the given monomial specifying-phrases:

$$-5 \oplus +3$$

So, we write and compute:

$$\begin{aligned}\left[+12x^{-5}\right] \otimes \left[+4x^{+3}\right] &= [+12 \otimes +4]\, x^{-5 \oplus +3} \\ &= +48x^{-2}\end{aligned}$$

(Since x is unspecified we cannot go any further.)

III-7. Identify $[+12x^{-5}] \otimes [-4x^{-3}]$

Discussion: In order to *multiply* monomial specifying-phrase with a common basis,

i. We get the *coefficient* of the resulting monomial specifying-phrase by *multiplying* the coefficients of the given monomial specifying-phrases:

$$+12 \otimes -4$$

ii. We get the *base* of the resulting monomial specifying-phrase by taking the base common to the given monomial specifying-phrases:

$$x$$

iii. We get the signed exponent of the resulting monomial specifying-phrase by "oplussing" the signed exponents of the given monomial specifying-phrases:

$$-5 \oplus -3$$

So, we write and compute:

$$\begin{aligned}\left[+12x^{-5}\right] \otimes \left[-4x^{+3}\right] &= [+12 \otimes -4]\, x^{-5 \oplus +3} \\ &= -48x^{-2}\end{aligned}$$

(Since x is unspecified we cannot go any further.)

III-8. Identify $[-12x^{+5}] \oplus\!\!\!\!\div [+4x^{+3}]$

Discussion: In order to *divide* monomial specifying-phrase with a common basis,

i. We get the *coefficient* of the resulting monomial specifying-phrase by *dividing* the coefficients of the given monomial specifying-phrases:

$$-12 \oplus\!\!\!\!\div + 4$$

ii. We get the *base* of the resulting monomial specifying-phrase by taking the base common to the given monomial specifying-phrases:

$$x$$

iii. We get the signed exponent of the resulting monomial specifying-phrase by "ominussing" the signed exponents of the given monomial specifying-phrases:

$$+5 \ominus +3$$

So, we write and compute:

$$\begin{aligned}\left[-12x^{+5}\right] \odiv \left[+4x^{+3}\right] &= [-12 \odiv + 4]\, x^{+5 \ominus +3} \\ &= [-12 \odiv + 4]\, x^{+5 \oplus -3} \\ &= -3x^{+2}\end{aligned}$$

(Since x is unspecified we cannot go any further.)

III-9. Identify $\left[+12x^{+5}\right] \odiv \left[-4x^{-3}\right]$

Discussion: In order to *divide* monomial specifying-phrase with a common basis,

i. We get the *coefficient* of the resulting monomial specifying-phrase by *dividing* the coefficients of the given monomial specifying-phrases:

$$+12 \odiv - 4$$

ii. We get the *base* of the resulting monomial specifying-phrase by taking the base common to the given monomial specifying-phrases:

$$x$$

iii. We get the signed exponent of the resulting monomial specifying-phrase by "ominussing" the signed exponents of the given monomial specifying-phrases:

$$+5 \ominus -3$$

So, we write and compute:

$$\begin{aligned}\left[+12x^{-5}\right] \odiv \left[-4x^{+3}\right] &= [+12 \odiv - 4]\, x^{+5 \ominus -3} \\ &= [+12 \odiv - 4]\, x^{+5 \oplus +3} \\ &= -3x^{+8}\end{aligned}$$

(Since x is unspecified we cannot go any further.)

III-10. Identify $\left[-12x^{-5}\right] \odiv \left[-4x^{+3}\right]$

Discussion: In order to *divide* monomial specifying-phrase with a common basis,

i. We get the *coefficient* of the resulting monomial specifying-phrase by *dividing* the coefficients of the given monomial specifying-phrases:

$$-12 \oplus\!\!\!\!\div - 4$$

ii. We get the *base* of the resulting monomial specifying-phrase by taking the base common to the given monomial specifying-phrases:

$$x$$

iii. We get the signed exponent of the resulting monomial specifying-phrase by "ominussing" the signed exponents of the given monomial specifying-phrases:

$$-5 \ominus +3$$

So, we write and compute:

$$\begin{aligned} \left[-12x^{-5}\right] \oplus\!\!\!\!\div \left[-4x^{+3}\right] &= [-12 \oplus\!\!\!\!\div - 4]\, x^{-5 \ominus +3} \\ &= [-12 \oplus\!\!\!\!\div - 4]\, x^{-5 \oplus -3} \\ &= +3x^{-8} \end{aligned}$$

(Since x is unspecified we cannot go any further.)

III-11. Identify $\left[-12x^{-5}\right] \oplus\!\!\!\!\div \left[+4x^{-3}\right]$

Discussion: In order to *divide* monomial specifying-phrase with a common basis,

i. We get the *coefficient* of the resulting monomial specifying-phrase by *dividing* the coefficients of the given monomial specifying-phrases:

$$-12 \oplus\!\!\!\!\div + 4$$

ii. We get the *base* of the resulting monomial specifying-phrase by taking the base common to the given monomial specifying-phrases:

$$x$$

iii. We get the signed exponent of the resulting monomial specifying-phrase by "ominussing" the signed exponents of the given monomial specifying-phrases:

$$-5 \ominus -3$$

So, we write and compute:

$$\begin{aligned} \left[-12x^{-5}\right] \oplus\!\!\!\!\div \left[+4x^{+3}\right] &= [-12 \oplus\!\!\!\!\div + 4]\, x^{-5 \ominus -3} \\ &= [-12 \oplus\!\!\!\!\div + 4]\, x^{-5 \oplus +3} \\ &= -3 \times x^{-2} \end{aligned}$$

(Since x is unspecified we cannot go any further.)

III-12. Identify the monomial specifying-phrase in **Dollars**: $\left[10 \times 2^{-5}\right] \times \left[6 \times 2^{+3}\right]$

Discussion: In order to *multiply* monomial specifying-phrase with a common basis,

i. We get the *coefficient* of the resulting monomial specifying-phrase by *multiplying* the coefficients of the given monomial specifying-phrases:

$$10 \times 6$$

ii. We get the *base* of the resulting monomial specifying-phrase by taking the base common to the given monomial specifying-phrases:

$$2$$

iii. We get the signed exponent of the resulting monomial specifying-phrase by "oplussing" the signed exponents of the given monomial specifying-phrases:

$$-5 \oplus +3$$

So, we write and compute:

$$\begin{aligned}\left[10 \times 2^{-5}\right] \times \left[6 \times 2^{+3}\right] &= [10 \times 6] \times 2^{-5 \oplus +3} \\ &= 60 \times 2^{-2} \\ &= \frac{60}{2 \times 2} \\ &= 15\end{aligned}$$

III-13. Identify the specifying-phrase in **Dollars**: $\left[512 \times 2^{-5}\right] \div \left[4 \times 2^{+3}\right]$

Discussion: In order to *divide* monomial specifying-phrase with a common basis,

i. We get the *coefficient* of the resulting monomial specifying-phrase by *dividing* the coefficients of the given monomial specifying-phrases:

$$512 \div 4$$

ii. We get the *base* of the resulting monomial specifying-phrase by taking the base common to the given monomial specifying-phrases:

$$2$$

iii. We get the signed exponent of the resulting monomial specifying-phrase by "ominussing" the signed exponents of the given monomial specifying-phrases:

$$-5 \ominus +3$$

So, we write and compute:

$$\left[512 \times 2^{-5}\right] \div \left[4 \times 2^{+3}\right] = [512 \div 4] \times 2^{-5 \ominus +3}$$
$$= [512 \div 4] \times 2^{-5 \oplus -3}$$
$$= 128 \times 2^{-8}$$
$$= \frac{128}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$$
$$= 0.5$$

III-14. Identify $\left[-3x^2 + 6x - 2 + x^{-1} - 2x^{-2}\right] \boxplus \left[4x^2 - x + 3 + 2x^{-1} - 3x^{-2}\right]$

Discussion: We write-in what "goes without saying" and line up vertically the monomials that can be *added* (because they have the same exponent):

$$\begin{array}{ll} & -3x^2 + 6x - 2 + 1x^{-1} - 2x^{-2} \\ \boxplus & +4x^2 - x + 3 + 2x^{-1} - 3x^{-2} \\ \hline & +x^2 + 5x + 1 + 1x^{-1} - 5x^{-2} \end{array}$$

III-15. Identify $\left[-3x^2 + 1 - 2x^{-1}\right] \boxplus \left[4x^3 - x^2 + 3x + 2 - 3x^{-1}\right]$

Discussion: We write-in what "goes without saying" and line up vertically the monomials that can be *added* (because they have the same exponent):

$$\begin{array}{ll} & -3x^2 + 0x + 1 - 2x^{-1} \\ \boxplus & + + 4x^3 - x^2 + 3x + 2 - 3x^{-1} \\ \hline & +4x^3 - 4x^2 + 3x + 4 - 5x^{-1} \end{array}$$

III-16. $\left[-3x + 6 - 2x^{-1} + x^{-2} - 2x^{-3}\right] \boxminus \left[4x - 1 + 3x^{-1} + 2x^{-2} - 3x^{-3}\right]$

Discussion:

- In order to *subtract* a polynomial, we *add the opposite of* this polynomial.
- We write-in what "goes without saying" and line up vertically the monomials that can be *added* (because they have the same exponent):

$$\begin{array}{ll} & -3x + 6 - 2x^{-1} + 1x^{-2} - 2x^{-3} \\ \boxplus & -4x + 1 - 3x^{-1} - 2x^{-2} + 3x^{-3} \end{array}$$

$$\overline{\phantom{-4x+1-3x^{-1}-2x^{-2}+3x^{-3}-5x^{-4}}}$$

$$-7x+7-5x^{-1}-x^{-2}+1x^{-3}$$

III-17. Identify $\left[-3+x^{-2}-2x^{-3}\right] \boxminus \left[4x-1+3x^{-1}+2x^{-2}-3x^{-3}+5x^{-4}\right]$

Discussion:

- In order to *subtract* a polynomial, we *add the opposite of* this polynomial.
- We write-in what "goes without saying" and line up vertically the monomials that can be *added* (because they have the same exponent):

$$\begin{array}{rr} & -3+0x^{-1}+x^{-2}-2x^{-3} \\ \boxplus & -4x+1-3x^{-1}-2x^{-2}+3x^{-3}-5x^{-4} \\ \hline & -4x-2-3x^{-1}-x^{-2}+x^{-3}-5x^{-4} \end{array}$$

III-18. Identify $-3x^2+x^{-1}-2x^{-2}+\left[4x^3-x+3+2x^{-1}-3x^{-2}\right]$

Discussion: We write-in what "goes without saying" and line up vertically the monomials that can be *added* (because they have the same exponent):

$$\begin{array}{rr} & -3x^2+0x+0+1x^{-1}-2x^{-2} \\ \boxplus & +4x^3+0x^2-x+3+2x^{-1}-3x^{-2} \\ \hline & +4x^3-3x^2-x+3+3x^{-1}-5x^{-2} \end{array}$$

III-19. Identify $5h^2+4h^3-6h^4+4h^5-\Big[-2h^3+h^4+7h^5-5h^6\Big]$

Discussion: "Subtracting" means "Adding the opposite". So,

$5h^2+4h^3-6h^4+4h^5-\Big[-2h^3+h^4+7h^5-5h^6\Big]$

means

$5h^2+4h^3-6h^4+4h^5+\Big[+2h^3-h^4-7h^5+5h^6\Big]$

We write-in what "goes without saying" and line up vertically the monomials that can be *added* (because they have the same exponent):

$$\begin{array}{rr} & +5h^2+4h^3-6h^4+4h^5 \\ \boxplus & +2h^3-h^4-7h^5+5h^6 \\ \hline \end{array}$$

$$+5h^2 + 6h^3 - 7h^4 - 3h^5 + 5h^6$$

III-20. Identify $\left[+5x - 6 + 7x^{-1}\right] \boxtimes [-3x + 5]$

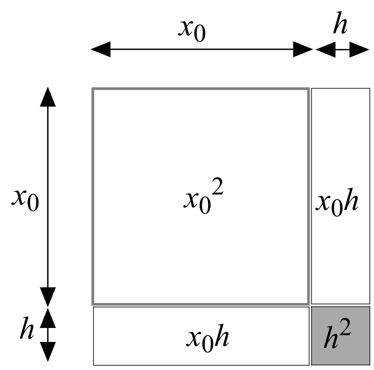
Discussion: Since polynomials are combinations of monomials, multiplication of polynomials is essentially multiplication of monomials. The additional step is to reduce the result by adding "like" monomials which is the reason for the layout we use.

$$\begin{array}{lr}
 & 5x - 6 + 7x^{-1} \\
\boxtimes & -3x + 5 \\
\hline
 & -15x^2 + 18x - 21 \\
 & +25x - 30 + 35x^{-1} \\
\hline
 & -15x^2 + 43x - 51 + 35x^{-1}
\end{array}$$

III-21. Identify $(-8 + h)^2$

Discussion: We can look at the question from two points of view (but we certainly should not multiply two copies of $-8 + h$).

- In a real-world situation, we are looking at the area of a square the "length" of whose side used to be -8 but which has now been increased by h to $-8+h$. In other words, we are looking at the following picture:



  We then get the area of the new square by adding to
  The original -8 by -8 square $= (-8)^2$:
  Two -8 by h strips $= 2 \cdot (-8)h$
  The little h by h square $= h^2$
- On paper, we get the binomial expansion as follows:

i. We construct the successive *powers*:
$(-8)^2h^0$
$(-8)^1h^1$
$(-8)^0h^2$
which we write horizontally:
$(-8)^2h^0 \quad (-8)^1h^1 \quad (-8)^0h^2$
ii. We write the successive *coefficients* with the aid of the Pascal Triangle:

$n := 0$			1		
$n := 1$		1		1	
$n := 2$	1		2		1

iii. We assemble the *powers* and the *coefficients*:
$1 \cdot (-8)^2h^0 \;+\; 2 \cdot (-8)^1h^1 \;+\; 1 \cdot (-8)^0h^2$

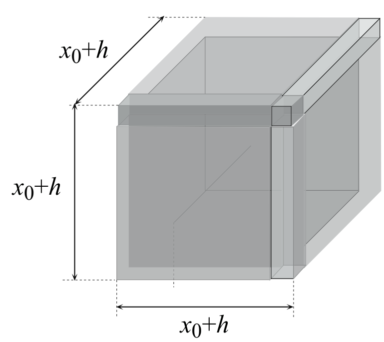
Either way, after computations, we end up with the binomial expansion

$$+64 - 16h + h^2$$

III-22. Identify $(-7+h)^3$

Discussion: We can look at the question from two points of view (but we certainly should *not* multiply three copies of $-7+h$).

- In a (pseudo) real-world situation, we are looking at the volume of a cube the "length" of whose side used to be -7 but which has now been increased by h to $-7+h$. In other words, we are looking at the following picture:



  We then get the volume of the new cube by adding to
  The original -7 by -7 by -7 cube $= (-7)^3$:
  Three -7 by -7 h-thick slabs $= 3 \cdot (-7)^2h$
  Three -7-long h by h rods $= 3 \cdot (-7)h^2$
  The little h by h by h cube $= h^3$
- On paper, we get the binomial expansion as follows:

i. We construct the successive *powers*:
$(-7)^3h^0$
$(-7)^2h^1$
$(-7)^1h^2$
$(-7)^0h^3$
which we write horizontally:
$(-7)^3h^0 \quad (-7)^2h^1 \quad (-7)^1h^2 \quad (-7)^0h^3$
ii. We write the successive *coefficients* with the help of the Pascal Triangle:

$$\begin{array}{lccccccc}
n := 0 & & & & 1 & & & \\
n := 1 & & & 1 & & 1 & & \\
n := 2 & & 1 & & 2 & & 1 & \\
n := 3 & 1 & & 3 & & 3 & & 1
\end{array}$$

iii. We assemble the *powers* and the *coefficients*:
$1 \cdot (-7)^3h^0 \; + \; 3 \cdot (-7)^2h^1 \; + \; 3 \cdot (-7)^1h^2 \; + \; 1 \cdot (-7)^0h^3$

Either way, after computations, we end up with the binomial expansion

$$-343 + 147h - 21h^2 + h^3$$

III-23. Approximate $\dfrac{6x^3 - x^2 + 13x - 6}{3x - 2}$ to x^{+1}.

Discussion: We divide $3x - 2$ into $6x^3 - x^2 + 13x - 6$:

$$\begin{array}{rl|llll}
 & & +2x^2 & +x & & \\ \hline
+3x & -2 \;) & +6x^3 & -x^2 & +13x & -6 \\
 & & -6x^3 & +4x^2 & & \\ \hline
 & & & +3x^2 & +13x &
\end{array}$$

We write the approximation:

$$\frac{6x^3 - x^2 + 13x - 6}{3x - 2} = +2x^2 + x + (...)$$

III-24. Approximate $\dfrac{9x^3 - 13x + 6}{-3x^2 - x + 2}$ to x^{-1}

Discussion: We divide $-3x^2 - x + 2$ into $9x^3 - 13x + 6$:

$$
\begin{array}{rrrrrr}
 & & -3x & +1 & +2x^{-1} & \\
\cline{3-6}
-3x^2 \; -x \; +2 & \big) & +9x^3 & +0x^2 & -13x & +6 \\
 & & -9x^3 & -3x^2 & +6x & \\
\cline{3-6}
 & & & -3x^2 & -7x & +6 \\
 & & & +3x^2 & +x & -2 \\
\cline{4-6}
 & & & & -6x & +4
\end{array}
$$

We write the approximation

$$\frac{9x^3 - 13x + 6}{-3x^2 - x + 2} = -3x + 1 + 2x^{-1} + [...]$$

III-25. Approximate $\dfrac{16 - 5h^2 + 7h^3}{4 - 3h + h^2}$ to h^1

Discussion: We divide $4 - 3h + h^2$ into $16 - 5h^2 + 7h^3$

$$
\begin{array}{rrrrrr}
 & & +4 & +2h & -5h^2 & \\
\cline{3-6}
+4 \; -3h \; +h^2 & \big) & +16 & +0h & -5h^2 & +7h^3 \\
 & & -16 & +12h & -4h^2 & \\
\cline{3-6}
 & & & +8h & -26h^2 & +10h^3 \\
 & & & -8h & +6h^2 & \\
\cline{4-6}
 & & & & -20h^2 & +10h^3
\end{array}
$$

We write the approximation:

$$\frac{8 + 2h - 26h^2 + 10h^3}{4 - 3h} = +4 + 2h - 5h^2 + [...]$$