

Chapter 10

Double Affine Problems

We conclude Part Two with double problems which are just like those in the preceding chapter but with *affine* problems instead of *basic* problems.

Conceptually, since *affine* problems can be reduced to *basic* problems, there will be absolutely nothing new in this chapter which serves only to show how much our investment in the PASCH PROCEDURE and the REDUCTION APPROACH will pay.

As a result, the only difficulty will be the “staying power” that will be required by the length of some of the computations.

EXAMPLE 10.1. Solve the double problem in Dollars

$$\text{BOTH} \begin{cases} +3x + 4.51 \leq +23.35 \\ +2.34 < +2x \end{cases}$$

1. The formula $+3x + 4.51 \leq +23.35$ is an *affine* inequation and the formula $+2.34 < +2x$ is a *basic* inequation so we should be able to find the solution subset on the basis of our previous work. At this point, though, we are not in a position to tell what “named” type of problem this is, if any.

2. We locate the *boundary* of the double problem by looking for the boundary point of each inequation, that is by solving the equation *associated* with each inequation.

a. The equation associated with the inequation $3x + 4.51 \leq +23.35$ is

$$+3x + 4.51 = +23.35$$

i. In order to reduce this *affine* equation to a *basic* equation, we must get rid of $+4.51$ on the right side which we do by adding its *opposite* -4.51 on both sides so as to be able to invoke the FAIRNESS THEOREM:

$$+3x + 4.51 - 4.51 = +23.35 - 4.51$$

$$+3x = +18.84$$

Then, dividing by +3 on both sides

$$+3x \div (+3) = +18.84 \div (+3)$$

gives the basic equation

$$x = +6.28$$

and therefore the boundary point +6.28.

ii. We check if the boundary point +6.28 is included or non-included in the solution subset.

Since we have

$$\begin{aligned} +3x + 4.51 \leq +23.35|_{x:=+6.28} & \text{ is TRUE} \\ +2.34 < +2x|_{x:=+6.28} & \text{ is TRUE} \end{aligned}$$

and since, in order for +6.28 to be a solution with the connector BOTH, +6.28 has to satisfy BOTH formulas, we have that

$$\text{BOTH} \begin{cases} +3x + 4.51 \leq +23.35|_{x:=+6.28} \\ +2.34 < +2x|_{x:=+6.28} \end{cases} \text{ is TRUE}$$

so that +6.28 is *included* in the solution subset and we must graph +6.28 with a *solid* dot.

b. The equation associated with the inequation $+2.34 < +2x$ is:

$$+2.34 = +2x$$

i. We reduce to a *basic* equation by dividing both sides by +2

$$x = +1.17$$

and therefore the boundary point is +1.17

ii. We check if the boundary point +1.17 is included or non-included in the solution subset.

Since we have

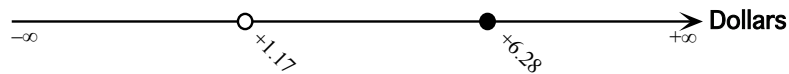
$$\begin{aligned} +3x + 4.51 \leq +23.35|_{x:=+1.17} & \text{ is TRUE} \\ +2.34 < +2x|_{x:=+1.17} & \text{ is FALSE} \end{aligned}$$

and since, in order for +1.17 to be a solution with the connector BOTH, +1.17 has to satisfy BOTH formulas, we have that

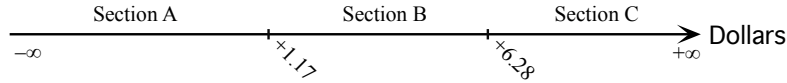
$$\text{BOTH} \begin{cases} +3x + 4.51 \leq +23.35|_{x:=+1.17} \\ +2.34 < +2x|_{x:=+1.17} \end{cases} \text{ is FALSE}$$

so that +1.17 is *non-included* in the solution subset and we must graph +6.28 with a *hollow* dot.

c. The *boundary* is



3. We locate the *interior* of the double problem by testing each one of the three sections determined by the two boundary points:



- We test Section A with, for instance, -1000 . That is, we must *evaluate* the two formulas in the given problem with -1000 .

$$\begin{aligned} +3x + 4.51 &\leq +23.35|_{x:=-1000} \\ +2.34 &< +2x|_{x:=-1000} \end{aligned}$$

that is

$$\begin{aligned} +3 \cdot (-1000) + 4.51 &\leq +23.35 \\ +2.34 &< +2 \cdot (-1000) \end{aligned}$$

that is

$$\begin{aligned} -3000 + 4.51 &\leq +23.35 \\ +2.34 &< -2000 \end{aligned}$$

that is

$$\begin{aligned} -2995.49 &\leq +23.35 \quad \text{which is TRUE} \\ +2.34 &< -2000 \quad \text{which is FALSE} \end{aligned}$$

Since, in order for -1000 to be a solution with the connector BOTH, -1000 has to satisfy BOTH formulas, we have that

$$\text{BOTH} \begin{cases} +3x + 4.51 \leq +23.35|_{x:=-1000} \\ +2.34 < +2x|_{x=-1000} \end{cases} \quad \text{is FALSE}$$

so that -1000 is *non-included* in the solution subset. **Pasch's Theorem** then tells us that all number-phrases in Section A are *non-included* in the solution subset.

- We test Section B with, for instance, $+2$. (We cannot test with 0 since 0 is not in Section B.) That is, we must *evaluate* the two formulas in the given problem with $+2$.

$$\begin{aligned} +3x + 4.51 &\leq +23.35|_{x:=+2} \\ +2.34 &< +2x|_{x:=+2} \end{aligned}$$

that is

$$\begin{aligned} +3 \cdot (+2) + 4.51 &\leq +23.35 \\ +2.34 &< +2 \cdot (+2) \end{aligned}$$

that is

$$\begin{aligned} +6 + 4.51 &\leq +23.35 \\ +2.34 &< +4 \end{aligned}$$

that is

$$\begin{aligned} +10.51 &\leq +23.35 \quad \text{which is TRUE} \\ +2.34 &< +4 \quad \text{which is TRUE} \end{aligned}$$

Since, in order for $+2$ to be a solution with the connector BOTH, $+2$ has to satisfy BOTH formulas, we have that

$$\text{BOTH} \begin{cases} +3x + 4.51 \leq +23.35|_{x:=+2} \\ +2.34 < +2x|_{x:=+2} \end{cases} \quad \text{is TRUE}$$

so that $+2$ is *included* in the solution subset. **Pasch's Theorem** then tells us that

- all number-phrases in Section B are *included* in the solution subset.
- We test Section C with, for instance, +1000. That is, we must *evaluate* the two formulas in the given problem with +1000.

$$\begin{aligned} +3x + 4.51 &\leq +23.35|_{x:=+1000} \\ +2.34 &< +2x|_{x:=+1000} \end{aligned}$$

that is

$$\begin{aligned} +3 \cdot (+1000) + 4.51 &\leq +23.35 \\ +2.34 &< +2 \cdot (+1000) \end{aligned}$$

that is

$$\begin{aligned} +3000 + 4.51 &\leq +23.35 \\ +2.34 &< +2000 \end{aligned}$$

that is

$$\begin{aligned} +3004.51 &\leq +23.35 && \text{which is FALSE} \\ +2.34 &< +2000 && \text{which is TRUE} \end{aligned}$$

Since, in order for +1000 to be a solution with the connector BOTH, +1000 has to satisfy BOTH formulas, we have that

$$\text{BOTH} \begin{cases} +3x + 4.51 \leq +23.35|_{x:=+1000} \\ +2.34 < +2x|_{x:=+1000} \end{cases} \text{ is FALSE}$$

so that -1000 is *non-included* in the solution subset. **Pasch's Theorem** then tells us that all number-phrases in Section A are *non-included* in the solution subset.

4. We represent and describe the solution subset of the problem in **Dollars**

$$\text{BOTH} \begin{cases} +3x + 4.51 \leq +23.35 \\ +2.34 < +2x \end{cases}$$

- The *graph* of the solution subset is



- The *name* of the solution subset is

$$(+1.17, +6.28] \text{ Dollars}$$