oriented items
orientation
opposite orientations
cancel

Chapter 2

Signed Number-Phrases

There are two issues with *plain* number phrases:

- With plain numerators, we can count *up* as far as we want but we cannot always count *down* as far as we want.
- Plain number phrases can only represent *collections* in which all the items are of *one* same kind but there are many situations in which items can come in either one of *two* flavors.

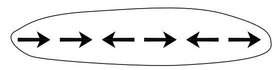
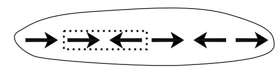
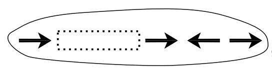
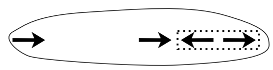
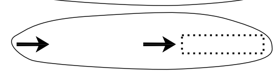
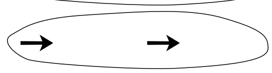
EXAMPLE 2.1.

- 3056.38 **Dollars** does not say if this was a *deposit* or a *withdrawal*,
- 37 800 **Dollars** does not say if a business is **in the red** (*owes* that money) or **in the black** (*has* that money).
- 62 **Dollars** does not say if a gambler is **ahead of the game** (has won more than s/he has lost) or **in the hole** (has lost more than s/he has won).
- 2 **Feet** from a benchmark does not say if a point is to the *left* or to the *right* of the benchmark.
- 5 **Inches** from the ground does not say if a point is *above* or *below* the ground.

2.1 Oriented Items

1. In the real world, there are many situations where we have to deal with collections of **oriented items**, that is items with *either one of two* **orientations** but where items with **opposite orientations cancel** each other so that collections of oriented items can only involve items that are all oriented the same way.

directed action
sided state
benchmark

Example 2.2. The collection  reduces automatically to only items with the same orientation: , , , , .

2. In the real-world, *oriented items* generally fall into either one of two categories:

- Items called **directed actions** which are "moves" of one kind or another but that can go either in *this-direction* or *that-direction*.

Example 2.3.
- a businesswoman may *deposit* three thousand dollar on a bank account or may *withdraw* three thousand dollars from a bank account.
- a gambler may *win* sixty-two dollars or may *lose* sixty-two dollars.
- on a horizontal line, a point can be moved two feet *foreward* or two feet *backwardward*
- on a vertical line, a point can be moved five inches *upward* or five inches *downward*

- Items called **sided states** which can be either on *this-side* or *that-side* of some **benchmark**.

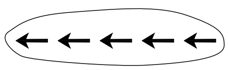
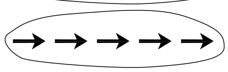
Example 2.4.
- a business may be three thousand dollars *in the red* or three thousand dollars *in the black*
- a gambler may be sixty-two dollars *ahead of the game* or sixty-two dollars *in the hole*.
- on a horizontal line with some benchmark, a point may be two feet *to the left* of the benchmark or two feet *to the right* of the benchmark
- on a vertical line with some baseline, a point may be five inches *above* the benchmark or five inches *below* the benchmark

2.2 Signed Number Phrases

signed numerator
size
quantitative
sign
qualitative
$+$
$-$
positive numerator

LANGUAGE 2.1. To make it clear which kind of numerator we are talking about, we will refer to the numerators which were introduced in **??** **??** as **plain numerators**.

To *represent* collections of oriented items on paper, we could of course just use *plain* numerators and *two* denominators, one for each orientation.

EXAMPLE 2.5. We could represent the collection of oriented items  by the plain number phrase 5 **Left Arrows** and the collection  by the plain number phrase 5 **Right Arrows**

However, even though orientation is arguably a *qualitative* matter, representing the *orientation* as part of the *numerator* instead of as part of the *denominator* will enormously facilitate *computations*.

1. Signed numerators consist of two parts:

- The **size** of a signed numerator, which is its **quantitative** part, is the *plain numerator* that specifies "how many" or "how much".

  LANGUAGE 2.2. Instead of the word "size", textbooks often use the word "absolute value" but we will *not* use this term in this text.[1]

- The **sign** of a signed numerator, which is its **qualitative** part, is the *symbol*, $+$ or $-$, that specifies "which way". The numerator 0 has no sign.

EXAMPLE 2.6. Say the signed number-phrase $+17.43$ **Dollars** specifies a *money transaction*. Then,

- Size $+17.43 = 17.43$ (*plain* number) specifies *how much* money was transacted,
- Sign $+17.43 = +$ specifies *which way* the money went.

2. Then:

Positive numerators are the numerators whose sign is $+$,

[1] Educologists will surely insist that we *should* conform. The reason we don't use the term "absolute value" is that, while "-5 is-larger-in-size-than $+2$" is a relation in $\mathbb{Z}$, $|-5| > |+2|$ is a relation in $\mathbb{N}$.

negative numerator
opposite
declare

Negative numerators are the numerators whose sign is $-$.

EXAMPLE 2.7. Positive Numerator: + 3 Negative Numerator: – 5

Sign
Size

Sign
Size

To make it always clear later what $+$ is being used for, we will use

> **AGREEMENT 2.1.** In this text, to make it easy to distinguish *positive* numerators from *plain* numerators, the $+$ *sign* will *never* "go without saying".

EXAMPLE 2.8. $+51$ is a *positive* numerator while 51 is a *plain* numerator.

3. The **opposite** of a signed numerator is the numerator with the same size and the other sign.

EXAMPLE 2.9.

The opposite of + 3 is – 3 The opposite of – 5 is + 5

Sign Sign Sign Sign
Size Size Size Size

If we want to write the opposite of a *negative* numerator which is a *positive* numerator, we *must* write the sign $+$ as, otherwise, by Agreement 2.1, without the sign $+$, the numerator will be seen as being *plain*.

EXAMPLE 2.10. If we want to talk about the opposite of -5, we *must* write $+5$ because if we just write 5 this will be seen as a *plain* numerator which is not the opposite of anything and has no opposite.

4. In order to *represent* on paper a real-world collection of oriented items, the first thing we need to do is to **declare**:

- which direction is to be represented by $+$. (And therefore which direction is to be represented by $-$,)
- which side of 0 is to be represented by $+$. (And therefore which side is to be represented by $-$.)

EXAMPLE 2.11. We *declare* that *right* steps are to be represented by $+$. (And therefore that that *left* steps are to be represented by $-$.)

signed number line

Then, to represent the collection $\leftarrow\leftarrow\leftarrow\leftarrow\leftarrow$ we will use the signed number phrase -5 **Arrows** and to represent the collection $\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow$ we will use the signed number phrase $+5$ **Arrows**

EXAMPLE 2.12. We declare that money *won* is to be represented by $+$. (And therefore that money *lost* is to be represented by $-$.)

When a *real-world* gambler:	We write on *paper*:
• *wins* sixty-two dollars	$+62$ **Dollars**
• *loses* sixty-two dollars	-62 **Dollars**

in which $+62$ is a *positive* signed-numerator and -62 is a *negative* signed-numerator.

EXAMPLE 2.13. We *declare* that accounts *in-the-black* are to be represented by *positive* numerators and that accounts *in-the-red* are to be represented by *negative* numerators and. Then,

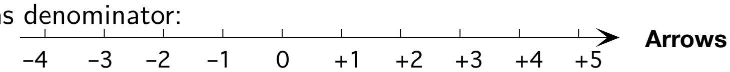
When a *real-world* business is:	We write on *paper*:
• three thousand dollars *in-the-black*	$+3000$ **Dollars**
• three thousand dollars *in-the-red*	-3000 **Dollars**

in which $+3000$ is a *positive* signed-numerator and -3000 is a *negative* signed-numerator.

2.3 Graphing Signed Number-Phrases

1. To *graph* signed number phrases, we use **signed number lines**.

EXAMPLE 2.14. Here is a signed number line for signed numerators with **Arrows** as denominator:



Just as with plain number phrases, we will use *solid dots* and *hollow dots* to graph signed number phrases.

2. From the *graphic* viewpoint:

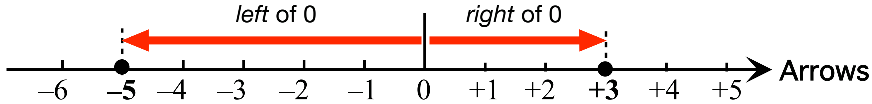
- The *sign* of a signed numerator codes *which side* of 0 the graph of the signed numerator is.

EXAMPLE 2.15. Since

Sign of -5 $= -$, the signed numerator -5 is left of 0.

Sign of $+3$ $= +$, the signed numerator $+3$ is right of 0.

So the graphs are:

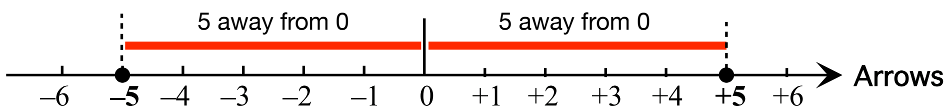


- The *size* of a signed numerator codes *how far away* from 0 the signed numerator is on a signed number line.

EXAMPLE 2.16. Since

Size of -5 is 5, the signed numerator -5 is 5 away from 0,

Size of $+5$ is 5, the signed numerator $+5$ is 5 away from 0.



?? **??** and section 2.2 Signed Number Phrases

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