Chapter 6

Single Translation & Dilation Problems

In **??**, we dealt with *basic* problems, that is with problems in which, given

- A *data set*,
- A *comparison formula* with a *gauge numerator*,

we were looking for the number-phrases in the data set that compared to the given *gauge numerator* according to the given *comparison formula*.

Much more often, though, we are interested in what an *agent of change* does to collections and so we compare the collections in the data set in their *final* state, that is *after* they have been changed by the agent.

So, in the paper world, a **reverse function problem** is a problem in which, given:

- A *data set*,
- A *function*,
- A *comparison formula* with a *gauge numerator*,

we are looking for the number-phrases in the data set whose *outputs* under the *function* compare to the *gauge numerator* according to the *comparison formula*.

reduce
undo

6.1 Single Translation Equation Problems

We solve translation equation problems by **reducing** the *translation* equation problem to a *basic* equation problem which we do by **undoing** the *translation* with the *opposite translation*.

EXAMPLE 6.1. Solve the addition *equation* problem

- *Data set*: $\underbrace{\{2, 3, 6, 7, 9\}}_{\text{Numerator Set}}$ $\underbrace{\text{Apples}}_{\text{Common Denominator}}$
- *Function*: $\underbrace{x}_{\text{Unspecified input}} \xrightarrow{+4} \underbrace{x + 4}_{\text{Output}}$
- *Comparison formula*: $\underbrace{x + 4}_{\text{Output}} = \underbrace{7}_{\text{Gauge}}$

In order to solve this *equation* problem,

i. We *reduce* the *addition* equation problem to a *basic* equation problem by undoing the translation $+4$ with the *opposite* translation -4:

$$\underbrace{x +4}_{\text{Output}} -4 =$$

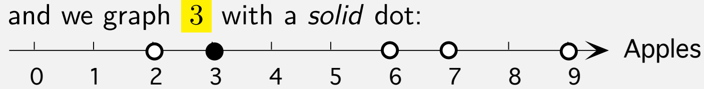
and to be fair we must also do

$$= \underbrace{7}_{\text{Gauge}} -4$$

The given *addition* equation problem will therefore have the same solution as the *basic* equation problem

$$x = 3$$

ii. We solve the *basic* equation as in **??**. Since the basic equation has the solution 3, the addition equation problem has the solution 3 too and we graph 3 with a *solid* dot:



EXAMPLE 6.2. Solve the *subtraction* equation problem

- *Data set*: $\underbrace{\{\text{-3, -2, +4, +7, +9}\}}_{\textit{Numerator Set}}$ $\underbrace{\textbf{Apples}}_{\textit{Common Denominator}}$
- *Function*: $\underbrace{x}_{\textit{Unspecified input}} \xrightarrow{\ominus -4} \underbrace{x \ominus -4}_{\textit{Output}}$
- *Comparison formula*: $\underbrace{x \ominus -4}_{\textit{Output}} \leqq \underbrace{+10}_{\textit{Gauge}}$

In order to solve this equation problem,

i. We *reduce* the *subtraction* equation problem to a *basic* equation problem by undoing the translation $\ominus -4$ with the *opposite* translation $\oplus -4$:

$$\underbrace{x \ominus -4}_{\text{Output}} \oplus -4 =$$

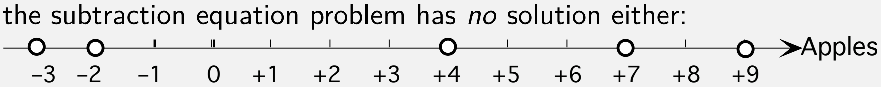
and to be fair we must also do

$$= \underbrace{+10}_{\text{Gauge}} \oplus -4$$

The given *subtraction* equation problem will therefore have the same solution as the *basic* equation problem

$$x = +6$$

ii. We solve the *basic* equation problem as in ??. Since $+6$ is *not* in the data set, the *basic* equation problem has no solution and therefore the subtraction equation problem has *no* solution either:



EXAMPLE 6.3. The data set is given to be all signed decimal numbers of **Dollars**. Solve the addition equation problem

$$x \oplus -24.32 = +32.67$$

i. We reduce the *addition* equation problem to a *basic* equation problem by undoing the translation $\oplus - 24.32$ with the *opposite translation* $\oplus + 24.32$:

$$x \oplus - 24.32 \oplus - 24.32 = +32.67 \oplus - 24.32$$

The given *translation* equation problem will therefore have the same solution as the *basic* equation problem

$$x = +8.35$$

ii. We solve the *basic* equation as in **??**. Since the basic equation has the solution $+8.35$, the translation equation has the solution $+8.35$ too and we graph $+8.35$ with a *solid* dot:

6.2 Single Dilation Equation Problems

In order to reduce a *dilation* equation problem to a *basic* equation problem, we *undo* the *dilation* with the *reciprocal dilation* which is the *reciprocal dilation*.

Example 6.4. The data set is given to be all signed decimal numbers of **Dollars**. Solve the dilation equation problem

$$x \otimes -24.32 = +32.67$$

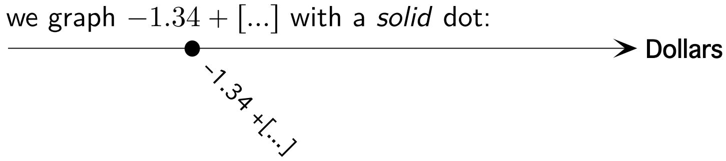
i. We reduce the *dilation* equation to a *basic* equation by undoing the dilation $\otimes - 24.32$ with the *reciprocal dilation* $\oplus\!\!\!\!\div - 24.32$:

$$x \otimes - 24.32 \oplus\!\!\!\!\div - 24.32 = +32.67 \oplus\!\!\!\!\div - 24.32$$

The given *translation* equation problem will therefore have the same solution as the *basic* equation problem

$$x = -1.34 + [...]$$

ii. We solve the *basic* equation as in **??**. Since the basic equation has the solution $-1.34 + [...]$, the translation equation has the solution $-1.34 + [...]$ too and we graph $-1.34 + [...]$ with a *solid* dot:



6.3 Single Translation Inequation Problems

locate
boundary
interior
test
interval

As with all *inequation* problems, we must use the PASCH PROCEDURE:

i. Locate the **boundaries** of the solution subset of the problem,

ii. Locate the **interior** of the solution subset of the problem by **testing** the resulting **intervals** and using the PASCH THEOREM.

EXAMPLE 6.5. Solve the translation *inequation* problem in **Dollars**

$$x \oplus -24.32 \leqq +32.67$$

We get the solution subset using the PASCH PROCEDURE:

1. Locating the Boundaries:

i. Solving the *associated equation problem*

$$x \oplus -24.32 = +32.67$$

as in Example 6.3 on page 3 gives the *boundary* $+56.99$.

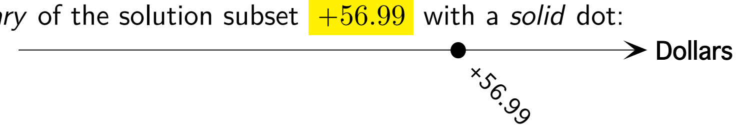
ii. To check if the *boundary* is in the solution subset we declare x to be $+56.99$ in the given *inequation*:

$$x \oplus -24.32 \leqq +32.67\big|_{x \leftarrow +56.99}$$

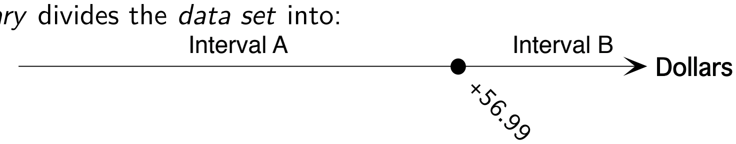
that is

$$+56.99 \oplus -24.32 \leqq +32.67$$
$$+32.67 \leqq +32.67$$

which is TRUE so that $+56.99$ *is* in the solution subset and we graph the *boundary* of the solution subset $+56.99$ with a *solid* dot:



2. Locating the Interior: We test each one of the two *intervals* that the *boundary* divides the *data set* into:



- To test Interval A, we declare x to be -1000 in the given *inequation*:

$$x \oplus -24.32 \leqq +32.67\big|_{x \leftarrow -1000}$$

that is

$$-1000 \oplus -24.32 \leqq +32.67$$
$$-1000 \oplus [...] \leqq -37.41$$

which is TRUE so that -1000 *is* in the solution subset and by the PASCH THEOREM the whole Interval A *is* part of the solution subset and therefore we graph Interval A with a *solid* bar:

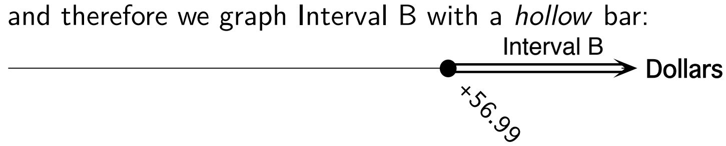
- To test Interval B, we declare x to be $+1000$ in the given *inequation*:

$$x \oplus -24.32 \leqq +32.67\big|_{x \leftarrow +1000}$$

that is

$$+1000 \oplus -24.32 \leqq +32.67$$
$$+1000 \oplus [...] \leqq -37.41$$

which is FALSE so that $+1000$ is *not* in the solution subset and by the PASCH THEOREM the whole Interval B is *not* part of the solution subset and therefore we graph Interval B with a *hollow* bar:



3. Altogether:

- The *teacher's graph* of the solution subset is:

- The *name* of the solution subset is:

$$\big(-\infty, +56.99\big)$$ **Dollars**

6.4 Single Dilation Inequation Problems

As with all *inequation* problems, we must use the PASCH PROCEDURE:

i. *Locate* the *boundaries* of the solution subset of the problem,

ii. *Locate* the *interior* of the solution subset of the problem by *testing* the resulting *intervals* and using the PASCH THEOREM.

EXAMPLE 6.6. Solve the dilation equation problem in **Dollars**

$$x \otimes -24.32 > +32.67$$

We get the solution subset using the PASCH PROCEDURE:

1. Locating the Boundaries:

i. Solving the *associated equation problem*

$$x \otimes -24.32 = +32.67$$

as in Example 6.4 on page 4 gives the *boundary* $-1.34 + [...]$.

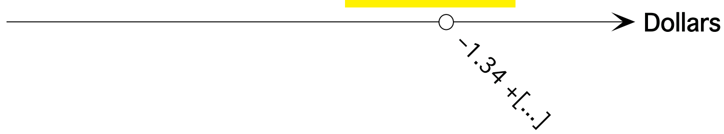
ii. To check if the *boundary* is in the solution subset we declare x to be $-1.34 + [...]$ in the given *inequation*:

$$x \otimes -24.32 > +32.67\big|_{x \leftarrow -1.34+[...]}$$

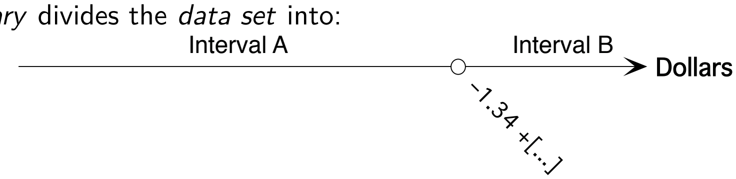
that is

$$-1.34 + [...] \otimes -24.32 > +32.67$$
$$+32.58 + [...] > +32.67$$

which is FALSE so that $-1.34 + [...]$ is *not* in the solution subset and we graph the *boundary* of the solution subset $-1.34 + [...]$ with a *hollow* dot:



2. Locating the Interior: We test each one of the two *intervals* that the *boundary* divides the *data set* into:



- To test Interval A, we declare x to be -1000 in the given *inequation*:

  $$x \otimes -24.32 > +32.67\big|_{x \leftarrow -1000}$$

  that is

  $$-1000 \otimes -24.32 > +32.67$$
  $$+24\,000 + [...] > +32.67$$

  which is TRUE so that -1000 *is* in the solution subset and by the PASCH THEOREM the whole Interval A *is* part of the solution subset and therefore we graph Interval A with a *solid* bar:

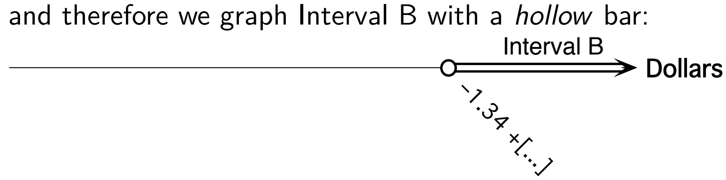
- To test Interval B, we declare x to be $+1000$ in the given *inequation*:

$$x \otimes -24.32 > +32.67|_{x \leftarrow +1000}$$

that is

$$+1000 \otimes -24.32 > +32.67$$
$$-24\,000 + [...] > +32.67$$

which is FALSE so that $+1000$ is *not* in the solution subset and by the PASCH THEOREM the whole Interval B is *not* part of the solution subset and therefore we graph Interval B with a *hollow* bar:



3. Altogether:

- The *teacher's graph* of the solution subset is:

- The *name* of the solution subset is:

$$\big(-\infty, -1.34 + [...]\big) \textbf{ Dollars}$$

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