Chapter 8

Double Basic Problems

Double Basic Equation Problems, 1 • Double Basic Inequation Problems, 5.

**Double problems** involve *two* formulas which can be either
• two equations
or
• two inequations
or, occasionally,
• one inequation and one equation.
that are *connected* by one of the following *connectors*:

- **Both**,
- **Either One Or Both**,
- **Either One But Not Both**

### 8.1 Double Basic Equation Problems

**Example 8.1.** Solve the double basic equation problem in **Dollars**

\[
\begin{align*}
\text{BOTH} & \begin{cases} 
    x = +32.67 \\
    x = -17.92 
\end{cases}
\end{align*}
\]

i. Since we are dealing with *equations*, the only numerators that *might* be a solution are the two *gauges*.
- We declare \( x \) to be **+32.67** in each one of the two equations:
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\[ x = +32.67 \quad \text{that is} \quad +32.67 = +32.67 \quad \text{which is TRUE} \]
\[ x = -17.92 \quad \text{that is} \quad -17.92 = -17.92 \quad \text{which is FALSE} \]

since \( \text{Both} \begin{cases} \text{TRUE} \\ \text{FALSE} \end{cases} \) is FALSE, \( +32.67 \) is not in the solution subset and we graph \( +32.67 \) with a hollow dot:

\[ \text{Dollars} \]

\[ x = +32.67 \quad \text{that is} \quad -17.92 = +32.67 \quad \text{which is FALSE} \]
\[ x = -17.92 \quad \text{that is} \quad -17.92 = -17.92 \quad \text{which is TRUE} \]

since \( \text{Either One Or Both} \begin{cases} \text{TRUE} \\ \text{FALSE} \end{cases} \) is FALSE, \( -17.92 \) is not in the solution subset and we graph \( -17.92 \) with a hollow dot:

\[ \text{Dollars} \]

ii. So the given double basic equation problem has no solution and:

• The teacher’s graph of the solution subset is

\[ \text{Dollars} \]

• The name of the solution subset is: \( \{ \} \) Dollars

EXAMPLE 8.2. Solve the double basic equation problem in Dollars

\[ \text{Either One Or Both} \begin{cases} x = +32.67 \\ x = -17.92 \end{cases} \]

i. Since we are dealing with equations, the only numerators that might be a solution are the two gauges.

• We declare \( x \) to be \( +32.67 \) in each one of the two equations:

\[ x = +32.67 \quad \text{that is} \quad +32.67 = +32.67 \quad \text{which is TRUE} \]
\[ x = -17.92 \quad \text{that is} \quad +32.67 = -17.92 \quad \text{which is FALSE} \]

and since \( \text{Either One Or Both} \begin{cases} \text{TRUE} \\ \text{FALSE} \end{cases} \) is TRUE, \( +32.67 \) is in the solution subset and we graph \( +32.67 \) with a solid dot:
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- We declare $x$ to be $-17.92$ in each one of the two equations:

\[
x = +32.67 \bigg|_{x=-17.92} \quad \text{that is} \quad -17.92 = +32.67 \quad \text{which is FALSE} \\
x = -17.92 \bigg|_{x=-17.92} \quad \text{which is TRUE}
\]

and since Either One Or Both \{FALSE is true, \footnotesize{-17.92} is in the solution subset and we graph \footnotesize{-17.92} with a solid dot:

ii. So the given double basic equation problem has the solutions \footnotesize{+32.67} and \footnotesize{-17.92} and:

- The teacher’s graph of the solution subset is

- The name of the solution subset is: \{ \footnotesize{-17.92, +32.67} \} Dollars

**EXAMPLE 8.3.** Solve the double basic equation problem in Dollars

Either One But Not Both \[
\begin{align*}
x &= +32.67 \\
x &= -17.92
\end{align*}
\]

i. Since we are dealing with equations, the only numerators that might be a solution are the two gauges.

- We declare $x$ to be \footnotesize{+32.67} in each one of the two equations:

\[
x = +32.67 \bigg|_{x=+32.67} \quad \text{that is} \quad +32.67 = +32.67 \quad \text{which is TRUE} \\
x = -17.92 \bigg|_{x=+32.67} \quad \text{which is FALSE}
\]

and, since Either One But Not Both \{TRUE is true, \footnotesize{+32.67} is in the solution subset and we graph \footnotesize{+32.67} with a solid dot:

- \footnotesize{+32.67} Dollars
Either One

- We declare \( x \) to be \(-17.92\) in each one of the two equations:
  \[
  x = +32.67 \bigg|_{x=-17.92} \\
  x = -17.92 \bigg|_{x=-17.92}
  \]
  that is \(-17.92 = +32.67\) which is false
  \(-17.92 = -17.92\) which is true

and, since Either One But Not Both \(\begin{cases} \text{false} \implies \text{true}, & -17.92 \text{ is} \\ \text{true} \implies \text{false} \end{cases}\) in the solution subset and we graph \(-17.92\) with a solid dot:

ii. So the given double basic equation problem has the solutions \(+32.67\) and \(-17.92\) and:
- The teacher’s graph of the solution subset is:
  \[
  \text{Dollars} \\
  -17.92 \quad +32.67
  \]
- The name of the solution subset is: \(\{-17.92, +32.67\} \text{ Dollars}\)

So, in the case of double basic equation problems, since we cannot have BOTH anyway, it makes no difference if the connector is Either One Or Both or if the connector is Either One But Not Both and we will just write \textbf{Either One}

\textbf{Example 8.4.} Solve the double basic equation problem in Dollars

\[
\text{Either One} \begin{cases} 
  x = +32.67 \\
  x = -17.92
\end{cases}
\]

i. Since we are dealing with equations, the only numerators that \textit{might} be a solution are the two gauges.
- We declare \( x \) to be \(+32.67\) in each one of the two equations:
  \[
  x = +32.67 \bigg|_{x=-32.67} \\
  x = -32.67 \bigg|_{x=-32.67}
  \]
  that is \(+32.67 = +32.67\) which is true
  \(+32.67 = -17.92\) which is false

and, since Either One \(\begin{cases} \text{true} \implies \text{true}, & +32.67 \text{ is} \\ \text{false} \implies \text{false} \end{cases}\) in the solution subset and we graph \(-17.92\) with a solid dot:
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We declare \( x \) to be \(-17.92\) in each one of the two equations:

\[
\begin{align*}
  x &= +32.67 \big| x = -17.92 \\
  x &= -17.92 \big| x = -17.92
\end{align*}
\]

that is \(-17.92 = +32.67\) which is FALSE

\(-17.92 = -17.92\) which is TRUE

and, since EITHER ONE \( \{\text{FALSE} \) is TRUE, \(-17.92\) is in the solution

subset and we graph \(-17.92\) with a solid dot:

\[\text{Dollars} \]

\(\bullet\) We declare \( x \) to be \(-17.92\) in each one of the two equations:

\[\text{Dollars} \]

\(\bullet\) The teacher’s graph of the solution subset is

\[\text{Dollars} \]

\(\bullet\) The name of the solution subset is: \( \{-17.92, +32.67\} \) Dollars

ii. So the given double basic equation problem has the solutions \(+32.67\) and

\(-17.92\) and:

- The teacher’s graph of the solution subset is

\[\text{Dollars} \]

\[\cdot \cdot \cdot \]

\[\text{Dollars} \]

\(\bullet\) The name of the solution subset is: \( \{-17.92, +32.67\} \) Dollars

8.2 Double Basic Inequation Problems

As we did with single problems, we will use the Pasch Procedure, that is we will:

i. Locate the boundary of the solution subset of the double problem,

ii. Locate the interior of the solution subset of the double problem

using test points and the PASCH THEOREM,

EXAMPLE 8.5. Solve the problem in Dollars

\[ \text{Both} \begin{cases} x > -37.41 \\ x < +68.92 \end{cases} \]

We get the solution subset using the Pasch Procedure:

1. Locating the Boundaries:

   i. Solving the associated double equation problem
Either One \[
\begin{align*}
    x &= -37.41 \\
    x &= +68.92
\end{align*}
\]
as in Example 8.4 on page 4 gives the boundaries \(-37.41\) and \(+68.92\).

\textbf{ii.} To check if the boundaries are in the solution subset:

- We declare \(x\) to be \(-37.41\) in the two given inequations:
  \[
  \begin{align*}
  x > -37.41 & \quad \text{that is} \quad -37.41 > -37.41 \quad \text{which is FALSE} \\
  x < +68.92 & \quad \text{that is} \quad -37.41 < +68.92 \quad \text{which is TRUE}
  \end{align*}
  \]
  and, since \textbf{both} \{false, true\} is FALSE, the boundary \(-37.41\) is not in the solution subset and we graph \(-37.41\) with a hollow dot:

- We declare \(x\) to be \(+68.92\) in the two given inequations:
  \[
  \begin{align*}
  x > -37.41 & \quad \text{that is} \quad +68.92 > -37.41 \quad \text{which is TRUE} \\
  x < +68.92 & \quad \text{that is} \quad +68.92 < +68.92 \quad \text{which is FALSE}
  \end{align*}
  \]
  and, since \textbf{both} \{true, false\} is FALSE, the boundary \(+68.92\) is not in the solution subset and we graph \(+68.92\) with a hollow dot:

So, the boundaries of the solution subset are:

\[
\begin{align*}
\text{-37.41 hollow dot} & \quad \text{+68.92 hollow dot}
\end{align*}
\]

\textbf{2. Locating the Interior:} We test each one of the three intervals that the boundaries divide the data set into:

- To test Interval A, we declare \(x\) to be \(-1000\) in the two given inequations:
  \[
  \begin{align*}
  x > -37.41 & \quad \text{that is} \quad -1000 > -37.41 \quad \text{which is FALSE} \\
  x < +68.92 & \quad \text{that is} \quad -1000 < +68.92 \quad \text{which is TRUE}
  \end{align*}
  \]
and, since BOTH \( \begin{cases} \text{FALSE} \\ \text{TRUE} \end{cases} \) is FALSE, the test point \(-1000\) is not in the solution subset and by the Pasch Theorem the whole Interval A is not in the solution subset and therefore we graph Interval A with a hollow bar:

- To test Interval B, we declare \( x \) to be \( 0 \) in the two given inequations:
  
  \[
  \begin{align*}
  x &> -37.41 \bigg|_{x=0} \\
  x &< +68.92 \bigg|_{x=0}
  \end{align*}
  \]

  that is \( 0 > -37.41 \) which is TRUE
  \( 0 < +68.92 \) which is TRUE

  and, since BOTH \( \begin{cases} \text{TRUE} \\ \text{TRUE} \end{cases} \) is true the test point \( 0 \) is in the solution subset and by the Pasch Theorem the whole Interval B is in the solution subset and therefore we graph Interval B with a solid bar:

- To test Interval C, we declare \( x \) to be \( +1000 \) in the two given inequations:

  \[
  \begin{align*}
  x &> -37.41 \bigg|_{x=+1000} \\
  x &< +68.92 \bigg|_{x=+1000}
  \end{align*}
  \]

  that is \( +1000 > -37.41 \) which is TRUE
  \( +1000 < +68.92 \) which is FALSE

  and, since BOTH \( \begin{cases} \text{TRUE} \\ \text{FALSE} \end{cases} \) is FALSE, the test point \( +1000 \) is not in the solution subset and by the Pasch Theorem the whole Interval C is not in the solution subset and therefore we graph Interval C with a hollow bar:

3. Altogether:

- The teacher’s graph of the solution subset is:
• The name of the solution subset is:

\((-37.41, +68.92)\) Dollars

**EXAMPLE 8.6.** Solve the problem in Dollars

\[
\text{Either One Or Both} \begin{cases} 
  x < -37.41 \\
  x \geq +68.92
\end{cases}
\]

We get the solution subset using the PASCH PROCEDURE:

1. **Locating the Boundaries:**
   i. Solving the associated equation problem

\[
\text{Either One} \begin{cases} 
  x = -37.41 \\
  x = +68.92
\end{cases}
\]

as in Example 8.4 on page 4 gives the boundaries \(-37.41\) and \(+68.92\).

   ii. To check if the boundaries are in the solution subset:

   • We declare \(x\) to be \(-37.41\) in the two given inequations:

\[
\begin{align*}
  x < -37.41 & |_{x = -37.41} \quad \text{that is} \quad -37.41 < -37.41 \quad \text{which is FALSE} \\
  x \geq +68.92 & |_{x = -37.41} \quad \text{that is} \quad -37.41 \geq +68.92 \quad \text{which is FALSE}
\end{align*}
\]

   and since \text{Either One Or Both} \begin{cases} 
  FALSE \\
  FALSE
\end{cases} is FALSE, the boundary \(-37.41\) is not in the solution subset and we graph \(-37.41\) with a hollow dot.

   • We declare \(x\) to be \(+68.92\) in the two given inequations:

\[
\begin{align*}
  x < -37.41 & |_{x = +68.92} \quad \text{that is} \quad +68.92 < -37.41 \quad \text{which is FALSE} \\
  x \geq +68.92 & |_{x = +68.92} \quad \text{that is} \quad +68.92 \geq +68.92 \quad \text{which is TRUE}
\end{align*}
\]

   and, since \text{Either One Or Both} \begin{cases} 
  TRUE \\
  TRUE
\end{cases} is TRUE, the boundary \(+68.92\) is in the solution subset and we graph \(+68.92\) with a solid dot.
2. Locating the Interior: We test each one of the three intervals that the boundaries divide the data set into:

- To test Interval A, we declare $x$ to be $-1000$ in the two given inequations:
  
  $$x \leq -37.41 \mid_{x=-1000} \quad \text{that is} \quad -1000 < -37.41 \quad \text{which is TRUE}$$
  $$x \geq +68.92 \mid_{x=-1000} \quad \text{that is} \quad -1000 \geq +68.92 \quad \text{which is FALSE}$$

  and, since Either One Or Both \{\begin{align*}
  \text{TRUE} & \quad \text{is true, the test point} \\
  \text{FALSE} & \quad \text{is false, the test point}
\end{align*}\} is true, the test point 

  $-1000$ is in the solution subset and by the Pasch Theorem the whole Interval A is in the solution subset and therefore we graph Interval A with a solid bar:

- To test Interval B, we declare $x$ to be $0$ in the two given inequations:
  
  $$x \leq -37.41 \mid_{x=0} \quad \text{that is} \quad 0 < -37.41 \quad \text{which is FALSE}$$
  $$x \geq +68.92 \mid_{x=0} \quad \text{that is} \quad 0 \geq +68.92 \quad \text{which is FALSE}$$

  and, since Either One Or Both \{\begin{align*}
  \text{FALSE} & \quad \text{is false, the test point} \\
  \text{FALSE} & \quad \text{is false, the test point}
\end{align*}\} is false, the test point 

  $0$ is not in the solution subset and by the Pasch Theorem the whole Interval B is not in the solution subset and therefore we graph Interval B with a hollow bar:
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• To test Interval C, we declare \( x \) to be \( +1000 \) in the two given inequations:

\[
\begin{align*}
  x &> -37.41 \big|_{x=-1000}^{+1000} \\
x &< +68.92 \big|_{x=-1000}^{+1000}
\end{align*}
\]

that is \( +1000 > -37.41 \) which is TRUE

\( +1000 < +68.92 \) which is FALSE

and since Either One Or Both \( \begin{cases} \text{TRUE} \\ \text{FALSE} \end{cases} \) is true the test point \( +1000 \) is in the solution subset and by the Pasch Theorem the whole Interval C is in the solution subset and therefore we graph Interval C with a solid bar:

3. Altogether:
   • The teacher’s graph of the solution subset is:

   -37.41 \( \longrightarrow \) +68.92 Dollars

   • The name of the solution subset is:

   \( ( -\infty, -37.41) \) Dollars \( \cup \) \( [ -37.41, +\infty) \) Dollars

EXAMPLE 8.7. Solve the problem in Dollars

Either One But Not Both
\[
\begin{cases} 
  x \leq -37.41 \\
  x < +68.92
\end{cases}
\]

We get the solution subset using the Pasch Procedure:

1. Locating the Boundaries:
   i. Solving the associated equation problem

   Either One
   \[
   \begin{cases} 
     x = -37.41 \\
     x = +68.92
   \end{cases}
   \]

   as in Example 8.4 on page 4 gives the boundaries \( -37.41 \) and \( +68.92 \).

   ii. To check if the boundaries are in the solution subset:

   • We declare \( x \) to be \( -37.41 \) in the two given inequations:

   \[
   \begin{align*}
   x &\leq -37.41 \big|_{x=-37.41} \\
x &< +68.92 \big|_{x=-37.41}
\end{align*}
\]

   that is \( -37.41 \leq -37.41 \) which is TRUE

   \( -37.41 < +68.92 \) which is TRUE
and since Either One But Not Both \( \begin{cases} \text{true} \\ \text{true} \end{cases} \) is true, the boundary \(-37.41\) is in the solution subset and we graph \(-37.41\) with a solid dot.

- We declare \( x \) to be \(+68.92\) in the two given inequations:

\[
\begin{align*}
  x &\leq -37.41 \quad \text{that is} \quad +68.92 \leq -37.41 \quad \text{which is FALSE} \\
  x &< +68.92 \quad \text{that is} \quad +68.92 < +68.92 \quad \text{which is FALSE}
\end{align*}
\]

and, since Either One But Not Both \( \begin{cases} \text{false} \\ \text{false} \end{cases} \) is false, the boundary \(+68.92\) is not in the solution subset and we graph \(+68.92\) with a hollow dot.

So, the boundaries of the solution subset are:

\[
\begin{array}{c}
\text{Dollars} \\
\text{–37.41} \\
\text{+68.92}
\end{array}
\]

2. Locating the Interior: We test each one of the three intervals that the boundaries divide the data set into:

\[
\begin{array}{c}
\text{Interval A} \\
\text{Interval B} \\
\text{Interval C} \\
\text{Dollars}
\end{array}
\]

- To test Interval A, we declare \( x \) to be \(-1000\) in the two given inequations:

\[
\begin{align*}
  x &\leq -37.41 \quad \text{that is} \quad -1000 \leq -37.41 \quad \text{which is TRUE} \\
  x &< +68.92 \quad \text{that is} \quad -1000 < +68.92 \quad \text{which is TRUE}
\end{align*}
\]

and since Either One But Not Both \( \begin{cases} \text{true} \\ \text{true} \end{cases} \) is false, the test point \(-1000\) is not in the solution subset and by the Pasch Theorem the whole Interval A is not in the solution subset and therefore we graph Interval A with a hollow bar:
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- To test Interval B, we declare \( x \) to be 0 in the two given inequations:

\[
\begin{align*}
  x & \leq -37.41 | x = 0 & \text{that is } & 0 \leq -37.41 & \text{which is FALSE} \\
  x & < +68.92 | x = 0 & \text{which is TRUE}
\end{align*}
\]

and since \textbf{Either One But Not Both} \( \left\{ \begin{array}{c}
\text{FALSE} \\
\text{TRUE}
\end{array} \right\} \) is true the test point 0 \( \text{is in the solution subset and by the Pasch Theorem the whole Interval B is in the solution subset and therefore we graph Interval B with a solid bar:} \)

- To test Interval C, we declare \( x \) to be 1000 in the two given inequations:

\[
\begin{align*}
  x & \leq -37.41 | x = 1000 & \text{that is } & 1000 \leq -37.41 & \text{which is FALSE} \\
  x & < +68.92 | x = 1000 & \text{which is FALSE}
\end{align*}
\]

and since \textbf{Either One But Not Both} \( \left\{ \begin{array}{c}
\text{FALSE} \\
\text{FALSE}
\end{array} \right\} \) is false, the test point 1000 \( \text{is not in the solution subset and by the Pasch Theorem the whole Interval C is not in the solution subset and therefore we graph Interval C with a hollow bar:} \)

3. Altogether:

- The \textit{teacher's graph} of the solution subset is:

- The \textit{name} of the solution subset is:

\( (-37.41, +68.92) \text{ Dollars} \)
**Example 8.8.** Solve the problem in Dollars

Either One But Not Both \(\begin{align*}
x &< -37.41 \\
x &= +68.92
\end{align*}\)

We get the solution subset using the Pasch Procedure:

1. **Locating the Boundaries:**
   i. Solving the associated equation problem

   \[
   \text{Either One } \begin{cases} 
   x = -37.41 \\
   x = +68.92
   \end{cases}
   \]

   as in Example 8.4 on page 4 gives the boundaries \(-37.41\) and \(+68.92\).

   ii. To check if the boundaries are in the solution subset:

   - We declare \(x\) to be \(-37.41\) in the two given inequations:

     \[
     \begin{align*}
     x &< -37.41 \\
     x &= +68.92
     \end{align*}
     \]

     that is \(-37.41 < -37.41\) which is FALSE

     and, since Either One But Not Both \(\begin{cases} 
     \text{FALSE} \\
     \text{FALSE}
     \end{cases}\) is FALSE, the boundary \(-37.41\) is not in the solution subset and we graph \(-37.41\) with a hollow dot.

   - We declare \(x\) to be \(+68.92\) in the two given inequations:

     \[
     \begin{align*}
     x &< -37.41 \\
     x &= +68.92
     \end{align*}
     \]

     that is \(+68.92 < -37.41\) which is FALSE

     and, since Either One But Not Both \(\begin{cases} 
     \text{FALSE} \\
     \text{TRUE}
     \end{cases}\) is TRUE, the boundary \(+68.92\) is in the solution subset and we graph \(+68.92\) with a solid dot.

So, the boundaries of the solution subset are:
2. Locating the Interior: We test each one of the three intervals that the boundaries divide the data set into:

- To test Interval A, we declare $x$ to be $-1000$ in the two given inequations:

$$
\begin{align*}
  x &< -37.41 \quad \text{that is} \quad -1000 < -37.41 \quad \text{which is TRUE} \\
  x &= +68.92 \quad \text{that is} \quad -1000 = +68.92 \quad \text{which is FALSE}
\end{align*}
$$

and since Either One But Not Both \{TRUE, FALSE\} is true the test point $-1000$ is in the solution subset and by the Pasch Theorem the whole Interval A is in the solution subset and therefore we graph Interval A with a solid bar.

- To test Interval B, we declare $x$ to be $0$ in the two given inequations:

$$
\begin{align*}
  x &< -37.41 \quad \text{that is} \quad 0 < -37.41 \quad \text{which is FALSE} \\
  x &= +68.92 \quad \text{that is} \quad 0 = +68.92 \quad \text{which is FALSE}
\end{align*}
$$

and since Either One But Not Both \{FALSE, FALSE\} is false, the test point $0$ is not in the solution subset and by the Pasch Theorem the whole Interval B is not in the solution subset and therefore we graph Interval B with a hollow bar.

- To test Interval C, we declare $x$ to be $+1000$ in the two given inequations:

$$
\begin{align*}
  x &< -37.41 \quad \text{that is} \quad +1000 < -37.41 \quad \text{which is FALSE} \\
  x &= +68.92 \quad \text{that is} \quad +1000 = +68.92 \quad \text{which is FALSE}
\end{align*}
$$
and since \textbf{Either One But Not Both} \begin{cases} \text{FALSE} & \text{is false, the test point} \\ \text{FALSE} & +1000 \end{cases} \text{ is not in the solution subset and by the Pasch Theorem the whole Interval } \mathcal{C} \text{ is not in the solution subset and therefore we graph Interval } \mathcal{C} \text{ with a hollow bar.}

3. Altogether:
- The \textit{teacher's graph} of the solution subset is:

- The \textit{name} of the solution subset is:
\[(\infty, -37.41) \cup \{ +68.92 \} \text{ Dollars}\]
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