Chapter 10

Multiplication - Division of Signed Numbers

Signed Co-multiplication, 1 • Signed Division, 4.

Just like multiplication and division of plain numbers were very different from addition and subtraction of plain numbers, multiplication and division of signed numerators are very different from addition and subtraction of signed numbers.

i. Multiplication of signed numbers cannot be repeatedoplussing.

**Example 10.1.** What repeated *oplussing* could $-3$ Arrows multiplied by $+5$ Arrows possibly stand for?

ii. Multiplication of signed numbers cannot give an area.

**Example 10.2.** What area could $-3$ Feet multiplied by $+5$ Feet possibly be? The denominator would of course have to be SquareFeet but what if the result of the multiplication turned out *negative*? What could a *negative area* be?

10.1 Signed Co-multiplication

This is where multiplication of signed numerators, for which we will use the symbol $\otimes$, is very useful.
1. We begin by looking at the real-world. As before, we want to investigate the change in a given state, gain or loss, that results from a given transaction, “in” or “out” as before but with oriented collections of “good” items or “bad” items.

**Example 10.3.** Consider a store where collections of apples can either get into the store or get out of the store. Moreover, the collections are collections of oriented items in that the apples can be either good apples—inasmuch as they will generate a profit when they are sold—or bad apples—inasmuch as they will generate a loss because they will have to be disposed of at a cost.

2. We now look at the way we will represent things on paper.

   a. We will represent
      - Collections of items getting “in” by positive number-phrases,
      - Collections of items getting “out” by negative number-phrases,

   **Example 10.4.** In Example 10.3, we would represent
   - Collections of apples getting into the store by positive number-phrases,
   - Collections of apples getting out of the store by negative number-phrases,

   b. We will represent
      - The unit-worth of “good” items by a positive number-phrase,
      - The unit-worth of “bad” items by negative number-phrases,

   **Example 10.5.** In Example 10.3, we would represent
   - The unit-value of good apples that is apples that will generate a sales profit of seven cents per apples by the co-number-phrase \(+7\) Cents Apple
   - The unit-value of bad apples that is apples that will generate a disposal cost of seven cents per apple by the co-number-phrase \(−7\) Cents Apple

3. We can now write the procedure for signed co-multiplication for which we will use the symbol \(⊗\):
   i. multiply the denominators (with cancellation).
   ii. Otime the numerators according to the way the result is a “good” change or a “bad” change:
      - A collection of “good” items getting “in” makes for a “good” change so
        \(+ ⊗ + = +\).
10.1. Signed Co-multiplication

**EXAMPLE 10.6.**
Three apples get *in* the store.
The apples have a unit-value of seven cents-per-apple gain.

The specifying phrase is

\[ [+3 \text{ Apples}] \otimes [+7 \text{ Cents \text{Apple}}] \]

We co-multiply

\[ [(+3) \otimes (+7)] \left[ \begin{array}{c} \text{Apples} \\ \otimes \end{array} \right] \begin{array}{c} \text{Cents} \\ \text{Apple} \end{array} \]

We get a twenty-one cent gain.

- A collection of “good” items getting “out” makes for a “bad” change so $+ \otimes - = -$.

**EXAMPLE 10.7.**
Three apples get *in* the store.
The apples have a unit-value of seven cents-per-apple loss.

The specifying phrase is

\[ [+3 \text{ Apples}] \otimes [-7 \text{ Cents \text{Apple}}] \]

We co-multiply

\[ [(+3) \otimes (-7)] \left[ \begin{array}{c} \text{Apples} \\ \otimes \end{array} \right] \begin{array}{c} \text{Cents} \\ \text{Apple} \end{array} \]

We get a twenty-one cent loss.

- A collection of “bad” items getting “in” makes for a “bad” change so $- \otimes + = -$.

**EXAMPLE 10.8.**
Three apples get *out* of the store.
The apples have a unit-value of seven cents-per-apple gain.

The specifying phrase is

\[ [-3 \text{ Apples}] \otimes [+7 \text{ Cents \text{Apple}}] \]

We co-multiply

\[ [(-3) \otimes (+7)] \left[ \begin{array}{c} \text{Apples} \\ \otimes \end{array} \right] \begin{array}{c} \text{Cents} \\ \text{Apple} \end{array} \]

We get a twenty-one cent loss.

- A collection of “bad” items getting “out” makes for a “good” change so $- \otimes - = +$.

**EXAMPLE 10.9.**
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Three apples get out of the store. The apples have a unit-value of seven cents-per-apple loss.
The specifying phrase is
We co-multiply
We get a twenty-one cent gain.

In other words, the rule for the multiplication of signs is:

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<th>+</th>
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<tr>
<td>+</td>
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<td>-</td>
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10.2 Signed Division

The rule of signs for the division of signs is the same as the rule for the multiplication of signs:

<table>
<thead>
<tr>
<th>+</th>
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<tbody>
<tr>
<td>+</td>
<td>+</td>
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<tr>
<td>-</td>
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</tbody>
</table>

**Example 10.10.** If we know that the store has incurred a twenty-one cents loss with three apples moving out, what was the unit-worth of the apples? Dividing $-21$ Cents by $-3$ Apples gives us $+7$ Cents
(Indeed, if apples moving out resulted in a loss, then the apples must have been good!)

**Example 10.11.** If we know that the store has incurred a twenty-one cents loss with good apples worth seven Cents each, how many apples moved and which way did they move?
Dividing $-21 \text{ Cents}$ by $+7 \frac{\text{Cents}}{\text{Apple}}$ gives us $-3 \text{ Apples}$ 
(Indeed, if $\text{good apples}$ moving resulted in a $\text{loss}$, then the $\text{apples}$ must have moved $\text{out}$!)

**Example 10.12.** If we know that the store has incurred a TWENTY-ONE $\text{cents}$ gain with THREE $\text{apples}$ moving out, what is the unit-worth of the $\text{apples}$?
Dividing $+21 \text{ Cents}$ by $-3 \text{ Apples}$ gives us $-7 \frac{\text{Cents}}{\text{Apple}}$
(Indeed, if $\text{apples}$ moving $\text{out}$ resulted in a $\text{gain}$, then the $\text{apples}$ must have been $\text{bad}$!)

**Example 10.13.** If we know that the store has incurred a TWENTY-ONE $\text{cents}$ gain with the unit-worth of the $\text{bad apples}$ worth SEVEN $\text{Cents}$ each, how many $\text{apples}$ moved and which way did they move?
Dividing $+21 \text{ Cents}$ by $-7 \frac{\text{Cents}}{\text{Apple}}$ gives us $-3 \text{ Apples}$
(Indeed, if $\text{bad apples}$ moving resulted in a $\text{gain}$, then the $\text{apples}$ must have moved $\text{out}$!)
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