Chapter 11

Coding Multiple Operations

(See http://www.devmathrevival.net/?p=2628#comment-204871)

Up until now, we followed the standard practice in the *sciences* and *technologies* in which numerators always come *with* denominators so that there never could be any doubt as to what was to be done. In this chapter, we will look at the difficulties created by the practice standard in *mathematics*, namely the use of numerators *without* denominators.

11.1 Plain numbers

1. With *plain* numbers, each symbol can only stand for *one* action.

a. With *plain* numbers, the symbol + can only stand for *add*.

EXAMPLE 11.1. In $2 + 3$, the symbol + stands for *add*.
So we count *up* $2 \xrightarrow{3,4,5} 5$, and the result is 5.

EXAMPLE 11.2. In $72.18 + 31.04$, + stands for *add*.

So we set up $\begin{array}{r} \overset{1}{} \\ 7\,2.1\,8 \\ +\quad 3\,1.0\,4 \\ \hline 1\,0\,3.2\,2 \end{array}$ and the result is 103.22.

b. With *plain* numbers, the symbol − can only stand for *subtract*.

EXAMPLE 11.3. In $5 - 3$, the symbol $-$ stands for *subtract*.
So we count *down* $5 \xrightarrow{4,3,2} 2$, and the result is 2.

EXAMPLE 11.4. In $3 - 5$, the symbol $-$ stands for *subtract*.
So we count *down* $3 \xrightarrow{2,1,0,?} ?$ and the subtraction cannot be done.

EXAMPLE 11.5. In $31.05 - 17.22$, the symbol $-$ stands for *subtract*.

We set up $\begin{array}{r} 3\,1.0\,5 \\ -\,1\,7.2\,2 \\ \hline 1\,3.8\,3 \end{array}$ and the result is 13.83.

EXAMPLE 11.6. In $17.22 - 31.05$, the symbol $-$ stands for *subtract*.
But we cannot do $17.22 - 31.05$.

c. With *plain* numbers, the symbol $\times$ can only stand for *multiply*.

EXAMPLE 11.7. In both 5×3 and 3×5, the symbol $\times$ stands for *multiply*.
Then the multiplication tables give us both $5 \times 3 = 15$ and $3 \times 5 = 15$

EXAMPLE 11.8. In 53.04×30.27, the symbol $\times$ stands for *multiply*.

We set up $\begin{array}{r} 5\,3.0\,4 \\ \times\,3\,0.2\,7 \\ \hline 3\,7\,1\,2\,8 \\ 1\,0\,6\,0\,8 \\ 1\,5\,9\,1\,2\,\cdot \\ \hline 1\,6\,0\,5.5\,2\,0\,8 \end{array}$ and the result is $1\,605.5208$

d. With *plain* numbers, the symbol $\div$ can only stand for *divide*.

EXAMPLE 11.9. In both $15 \div 3$ and $17 \div 5$, the symbol $\times$ stands for *divide*.
Then the multiplication tables give us both $15 \div 3 = 5$ and $17 \div 5 = 3$ with a remainder of 2

EXAMPLE 11.10. In $523.14 \div 32.07$, the symbol $\div$ stands for *divide*.

We set up $\begin{array}{r} 3 \\ 32.1\,\overline{)\,98.7} \\ 96.3 \\ \hline 2.4 \end{array}$ and the result is $3 + [...]$

2. Generally, we *operate* the same way we read and write, that is from left to right. However this may not be the case when using *more than one* operation and we then need to think again about *denominators*.

EXAMPLE 11.11. $7 + 4 + 2$ can only come from something like 7 **Apples** $+$ 4 **Apples** $+$ 2 **Apples** and so we do:

$$\begin{aligned} 7 + 4 + 2 \\ 11 + 2 \\ 13 \end{aligned}$$

EXAMPLE 11.12. $7 - 4 - 2$ can only come from something like 7 **Apples** $-$ 4 **Apples** $-$ 2 **Apples** and so we do, *if we can*:

$$\begin{aligned} 7 - 4 - 2 \\ 3 - 2 \\ 1 \end{aligned}$$

EXAMPLE 11.13. $7 - 5 - 3$ can only come from something like 7 **Apples** $-$ 5 **Apples** $-$ 3 **Apples** which we cannot do because 7 **Apples** $-$ 5 **Apples** $=$ 2 **Apples** and we cannot do 2 **Apples** $-$ 3 **Apples**

EXAMPLE 11.14. $2 + 3 \times 5$ can only come from either:

- Something like 2 **SqFeet** $+$ 3 **Feet** $\times$ 5 **Feet**

or from

- Something like 2 **Dollars** $+$ 3 **Apples** $\times$ $5 \frac{\textbf{Dollars}}{\textbf{Apple}}$

Either way, if we try to do the addition first we are looking at

- 2 **SqFeet** $+$ 3 **Feet**

or

- 2 **Dollars** $+$ 3 **Apples**

which make no sense.

If we try to do the *multiplication first* we are looking at:

- 3 **Feet** $\times$ 5 **Feet** $=$ 15 **SqFeet**

and

- 3 **Apples** $\times$ $5 \frac{\textbf{Dollars}}{\textbf{Apple}}$ $=$ 15 **Dollars**

which make perfect sense. So, altogether, we do:

parentheses

- $2\,\textsf{SqFeet} + 3\,\textsf{Feet} \times 5\,\textsf{Feet} = 2\,\textsf{SqFeet} + 15\,\textsf{SqFeet} = 17\,\textsf{SqFeet}$
- $2\,\textsf{Dollars} + 3\,\textsf{Apples} \times 5\frac{\textsf{Dollars}}{\textsf{Apple}} = 2\,\textsf{Dollars} + 15\,\textsf{Dollars} = 17\,\textsf{Dollars}$

So, even without denominators, we *must* do the *multiplication first*:

$$2 + 3 \times 5 = 2 + 15 = 17$$

3. Same with *division*: we must do division ahead of addition and subtraction.

4. If, for whatever reason, somebody wants an opearation to be done first, this person *must* say so by using **parentheses**.

Example 11.15. In $(2 + 3) \times 5$, the *parentheses* say we *must* do the addition first:

$$(2 + 3) \times 7 = 5 \times 7 = 35$$

11.2 Signed numbers

With *signed* numbers, things get complicated because each symbol can now have *several* meanings and because even when dealing with *signed* numbers, we still need to use *plain* operations for the *sizes* of these signed numbers.

- The symbol $+$ can now mean:
  positive
  signed addition (for which, until now, we used the symbol $\oplus$)
  plain addition (Still used for the *sizes* of signed numbers.)
- The symbol $-$ can now mean:
  negative
  signed subtraction (for which, until now, we used the symbol $\ominus$)
  plain subtraction (Still used for the *sizes* of signed numbers.)
- The symbol $\times$ can now mean:
  signed multiplication (for which, until now, we used the symbol $\otimes$)
  plain multiplication (Still used for the *sizes* of signed numbers.)

  The symbol $\div$ can now mean:
  signed division (for which, until now, we used the symbol $\oplus\!\!\!\!\div$)
  plain division (Still used for the *sizes* of signed numbers.).

It was to make things easier that until now we used the symbols $\oplus$, $\ominus$. $\otimes$, and $\oplus\!\!\!\!\div$ for the *signed* operations. We will now see how to use just $+$, $-$, $\times$, and $\div$.

1. The first difficulty is that writers often let the *positive symbol go without saying*.

EXAMPLE 11.16. $7 - (-3)$ stands for $+7 \ominus -3$ and therefore we do $+7 \oplus +3 = +10$

2. The second difficulty comes from the fact that writers often let *both* the *positive symbol* and the *signed addition symbol* go without saying.

EXAMPLE 11.17. $3 - 7$
i. If we are sure that the writer intended *plain numbers*, then $3 - 7$ can't be done
ii. If we are sure that the writer intended *signed numbers* then $3 - 7$ can stand for either:

- $+3 \oplus -7$ if the writer intended the $\oplus$ to go without saying and therefore the $-$ to be the sign going with the 7

or

- $+3 \ominus +7$ if the writer intended the $-$ to mean $\ominus$ and the sign of 7 to go without saying therefore to be $+$.

But, since $\ominus$ is done as $\oplus$ *opposite*, both compute to -4 and which the writer intended does not matter.

EXAMPLE 11.18. With $-4 \cdot 6 - 7$ we know the writer intended *signed numbers* because of the -4.
But then the writer intended $-4 \otimes +6 \oplus -7$.
So, we do the *multiplication first*:

$$-4 \otimes +6 \oplus -7 =$$

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