Chapter 11

Coding Multiple Operations

Plain numbers, 1 • Signed numbers, 4.

(See http://www.devmathrevival.net/?p=2628#comment-204871)

Up until now, we followed the standard practice in the sciences and technologies in which numerators always come with denominators so that there never could be any doubt as to what was to be done. In this chapter, we will look at the difficulties created by the practice standard in mathematics, namely the use of numerators without denominators.

11.1 Plain numbers

1. With plain numbers, each symbol can only stand for one action.

a. With plain numbers, the symbol + can only stand for add.

EXAMPLE 11.1. In $2 + 3$, the symbol + stands for add.

So we count up $2 \rightarrow 3 \rightarrow 4 \rightarrow 5$, and the result is 5.

EXAMPLE 11.2. In $72.18 + 31.04$, + stands for add.

So we set up $7 \rightarrow 2 \rightarrow 1 \rightarrow 8$ and the result is 103.22.

b. With plain numbers, the symbol − can only stand for subtract.
EXAMPLE 11.3. In $5 - 3$, the symbol $\hat{\_}$ stands for *subtract*. So we count *down* $5 \rightarrow 2$, and the result is $2$.

EXAMPLE 11.4. In $3 - 5$, the symbol $\hat{\_}$ stands for *subtract*. So we count *down* $3 \rightarrow 0, ?$ and the subtraction cannot be done.

EXAMPLE 11.5. In $31.05 - 17.22$, the symbol $\hat{\_}$ stands for *subtract*. We set up $\begin{array}{c} 31.05 \\ \hline 17.22 \end{array}$ and the result is $13.83$.

EXAMPLE 11.6. In $17.22 - 31.05$, the symbol $\hat{\_}$ stands for *subtract*. But we cannot do $17.22 - 31.05$.

c. With *plain* numbers, the symbol $\times$ can only stand for *multiply*.

EXAMPLE 11.7. In both $5 \times 3$ and $3 \times 5$, the symbol $\times$ stands for *multiply*. Then the multiplication tables give us both $5 \times 3 = 15$ and $3 \times 5 = 15$.

EXAMPLE 11.8. In $53.04 \times 30.27$, the symbol $\times$ stands for *multiply*. We set up $\begin{array}{c} 53.04 \\ \times 30.27 \\ \hline 1605.5208 \end{array}$ and the result is $1605.5208$.

d. With *plain* numbers, the symbol $\div$ can only stand for *divide*.

EXAMPLE 11.9. In both $15 \div 3$ and $17 \div 5$, the symbol $\div$ stands for *divide*. Then the multiplication tables give us both $15 \div 3 = 5$ and $17 \div 5 = 3$ with a remainder of $2$.

EXAMPLE 11.10. In $523.14 \div 32.07$, the symbol $\div$ stands for *divide*. We set up $\begin{array}{c} \phantom{3} \underline{32.07} \\ 98.7 \\ \hline 96.3 \\ \hline 2.4 \end{array}$ and the result is $3 + [...]$. 
2. Generally, we operate the same way we read and write, that is from left to right. However this may not be the case when using more than one operation and we then need to think again about denominators.

**Example 11.11.** \(7 + 4 + 2\) can only come from something like 7 Apples + 4 Apples + 2 Apples and so we do:

\[
\begin{align*}
7 + 4 + 2 &= 11 + 2 \\
11 + 2 &= 13
\end{align*}
\]

**Example 11.12.** \(7 - 4 - 2\) can only come from something like 7 Apples - 4 Apples - 2 Apples and so we do, if we can:

\[
\begin{align*}
7 - 4 - 2 &= 3 - 2 \\
3 - 2 &= 1
\end{align*}
\]

**Example 11.13.** \(7 - 5 - 3\) can only come from something like 7 Apples - 5 Apples - 3 Apples which we cannot do because 7 Apples - 5 Apples = 2 Apples and we cannot do 2 Apples - 3 Apples.

**Example 11.14.** \(2 + 3 \times 5\) can only come from either:
- Something like 2 SqFeet + 3 Feet \(\times\) 5 Feet

or from
- Something like 2 Dollars + 3 Apples \(\times\) \(\frac{5\text{Dollars}}{\text{Apple}}\)

Either way, if we try to do the addition first we are looking at
- 2 SqFeet + 3 Feet

or
- 2 Dollars + 3 Apples

which make no sense.

If we try to do the multiplication first we are looking at:
- 3 Feet \(\times\) 5 Feet = 15 SqFeet

and
- 3 Apples \(\times\) \(\frac{5\text{Dollars}}{\text{Apple}}\) = 15 Dollars

which make perfect sense. So, altogether, we do:
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Parentheses

- \(2 \text{SqFeet} + 3 \text{Feet} \times 5 \text{Feet} = 2 \text{SqFeet} + 15 \text{SqFeet} = 17 \text{SqFeet}\)
- \(2 \text{Dollars} + 3 \text{Apples} \times 5 \frac{\text{Dollars}}{\text{Apple}} = 2 \text{Dollars} + 15 \text{Dollars} = 17 \text{Dollars}\)

So, even without denominators, we must do the multiplication first:

\[2 + 3 \times 5 = 2 + 15 = 17\]

3. Same with division: we must do division ahead of addition and subtraction.

4. If, for whatever reason, somebody wants an operation to be done first, this person must say so by using parentheses.

**Example 11.15.** In \((2 + 3) \times 5\), the parentheses say we must do the addition first:

\[(2 + 3) \times 7 = 5 \times 7 = 35\]

11.2 Signed numbers

With signed numbers, things get complicated because each symbol can now have several meanings and because even when dealing with signed numbers, we still need to use plain operations for the sizes of these signed numbers.

- The symbol + can now mean:
  - positive
  - signed addition (for which, until now, we used the symbol \(\oplus\))
  - plain addition (Still used for the sizes of signed numbers.)
- The symbol − can now mean:
  - negative
  - signed subtraction (for which, until now, we used the symbol \(\ominus\))
  - plain subtraction (Still used for the sizes of signed numbers.)
- The symbol \(\times\) can now mean:
  - signed multiplication (for which, until now, we used the symbol \(\otimes\))
  - plain multiplication (Still used for the sizes of signed numbers.)

The symbol \(\div\) can now mean:

- signed division (for which, until now, we used the symbol \(\oslash\))
- plain division (Still used for the sizes of signed numbers.)

It was to make things easier that until now we used the symbols \(\oplus, \ominus, \otimes, \oslash\), and \(\oplus\) for the signed operations. We will now see how to use just +, −, \times, and \(\div\).
11.2. Signed numbers

1. The first difficulty is that writers often let the positive symbol go without saying.

**Example 11.16.** $7 - (-3)$ stands for $+7 \oplus -3$ and therefore we do $+7 \oplus +3 = +10$

2. The second difficulty comes from the fact that writers often let both the positive symbol and the signed addition symbol go without saying.

**Example 11.17.** $3 - 7$

i. If we are sure that the writer intended *plain numbers*, then $3 - 7$ can’t be done.

ii. If we are sure that the writer intended *signed numbers* then $3 - 7$ can stand for either:

- $+3 \oplus -7$ if the writer intended the $\oplus$ to go without saying and therefore the $-$ to be the sign going with the 7

or

- $+3 \ominus +7$ if the writer intended the $-$ to mean $\ominus$ and the sign of 7 to go without saying therefore to be +.

But, since $\ominus$ is done as $\oplus$ *opposite*, both compute to $-4$ and which the writer intended does not matter.

**Example 11.18.** With $-4 \cdot 6 - 7$ we know the writer intended *signed numbers* because of the $-4$.

But then the writer intended $-4 \otimes +6 \oplus -7$.

So, we do the *multiplication first*:

$$-4 \otimes +6 \oplus -7 =$$
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