take-from collection of *oriented* items

Chapter 9

Signed Subtracting

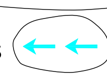
Detaching Collections of Oriented Items, 1 • Subtracting Signed Number Phrases, 3 • Bank Accounts, 6.

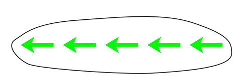
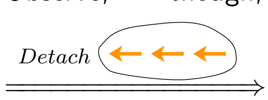
9.1 Detaching Collections of Oriented Items

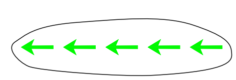
1. Detaching a **take-from collection of *oriented* items** from an initial collection of *oriented* items is quite different from detaching a take-from collection of *plain* items from an initial collection of *plain* items.

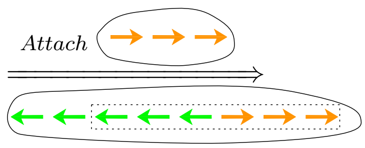
- When the orientation of the items in the take-from collection is *the same as* the orientation of the items in the initial collection, up-front, the detachment process still looks pretty much the same as with collections of plain items.
  - If the take-from collection is *smaller in size* than the initial collection, things work exactly as with collection of plain items.

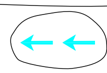
EXAMPLE 9.1. 

and the *final collection* is .

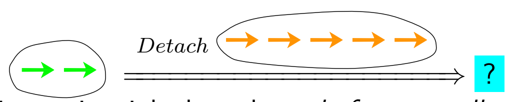
Observe, though, that instead of  

we could have done 

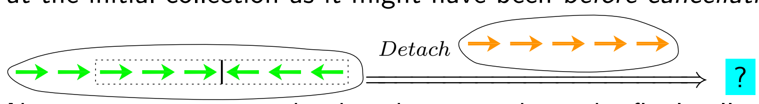


with, after cancellations, the same final collection .

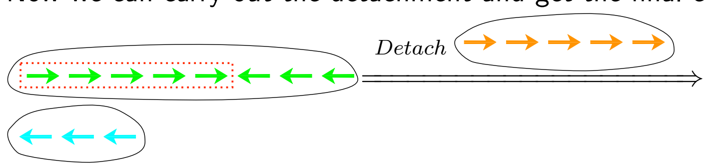
– If the take-from collection is *larger in size* than the initial collection, up front, we cannot carry out the detachment and we must look at the initial collection as it might have been *before cancellations*. We can then detach the items in the take-from collection from the items in the initial collection with the same orientation.

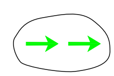
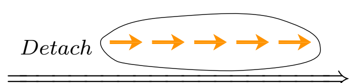
EXAMPLE 9.2. 

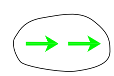
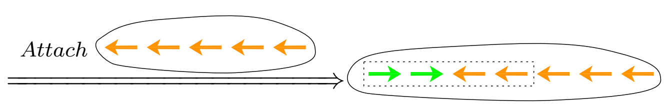
We look at the initial collection as it might have been *before cancellation*:

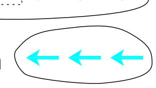


Now we can carry out the detachment and get the final collection:



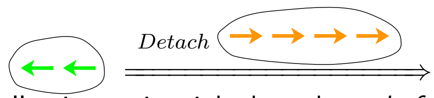
Observe, though, that instead of  

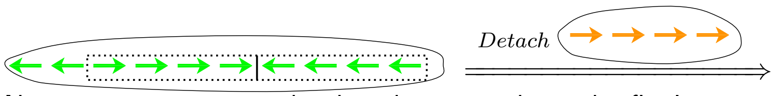
we could have done  

with, after cancellations, the same final collection .

- When the orientation of the items in the take-from collection is *the opposite of* the orientation of the items in the initial collection, things are totally unlike the plain case and we must look at the initial collection as it might have been *before cancellations*

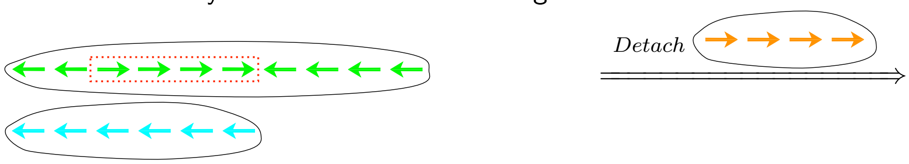
function
input-output rule
unspecified input
specific inputs
function name
output specifying code
specific outputs

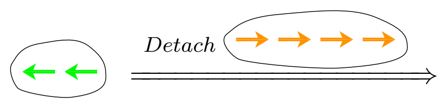
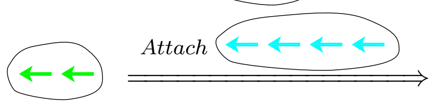
Example 9.3.  ? We look at the initial state of the collection as it might have been *before cancellations*.

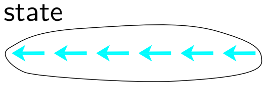


?

Now we can carry out the detachment and get the final state:



Observe, though, that instead of  we could have done  with the same final state

 .

2. Indeed, as we saw in the above **Examples**, in the real world, instead of detaching a take-from collection of oriented items, we can always attach an add-on collection of items with the opposite orientation because the oriented items in the add-on collection will cancel the oriented items in the initial collection.

9.2 Subtracting Signed Number Phrases

1. We saw in Chapter 5 that real world *agents of change* are represented on paper by **functions** which we specify with an **input-output rule** that consists of:
i. An **unspecified input** eventually to be replaced by **specific inputs**, that is the number phrases that represent the *initial collections*.
ii. A **function name**, that is the name of the function that represents the agent of change
iii. The **output specifying code** which is the code that specifies the *output* of the function in terms of the input. The **specific outputs** are the number phrases that represent the final collections.
Thus, the real world *action*

plain subtracting
⊖
ominussing
signed subtracting function
signed subtraction

Initial collection $\xrightarrow{\text{Agent of Change}}$ Final collection

is represented on paper by the *input-output rule*

Unspecified input $\xrightarrow{\text{Name of function}}$ Output-specifying code

To be replaced by specific inputs — Specifies what to do with the inputs to get the outputs

2. It will be most important to know what we are talking about:

LANGUAGE 9.1. (Plain versus Signed Subtracting) To make it clear which *subtracting* we are talking about, we will refer to the subtracting of plain numerators which were introduced in Chapter 5 as **plain subtracting**.
In fact, to keep things clear, we will use for signed subtracting the symbol ⊖, read "ominus", and, in the future, instead of saying that we are "*subtracting* a signed numerator", we will say that we are "**ominussing** that numerator".

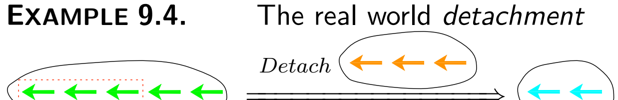
3. Just as with plain subtraction, we represent the action of detaching a take-from collection of *oriented* items from a collection of *oriented* items by the input-output rule of a **signed subtracting function**, often called **signed subtraction** for short.

Input signed numerator $\xrightarrow{\ominus \text{ signed numerator}}$ Output numerator =
Input signed numerator ⊖ *signed* numerator

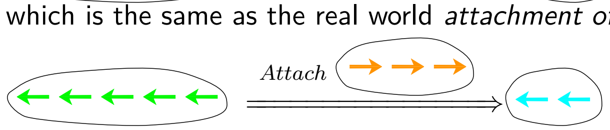
4. But, while the *addition* of two signed number phrases was a bit complicated, the *subtraction* of two signed number phrases is quite simple:

THEOREM 9.1 Ominus To ominus a number phrase, oplus the opposite of the number phrase

EXAMPLE 9.4. The real world *detachment*



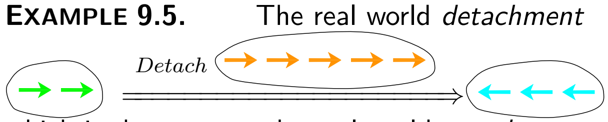
which is the same as the real world *attachment of the opposite*



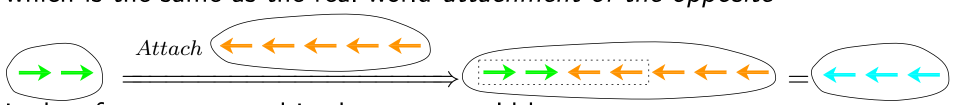
is therefore represented in the paper world by

$$\begin{aligned} -5\,\text{Arrows} \xrightarrow{\ominus -3\,\text{Arrows}}\; & -5\,\text{Arrows} \ominus -3\,\text{Arrows} \\ &= -5\,\text{Arrows} \oplus +3\,\text{Arrows} \\ &= -(5-3)\,\text{Arrows} \\ &= -2\,\text{Arrows} \end{aligned}$$

Example 9.5. The real world *detachment*



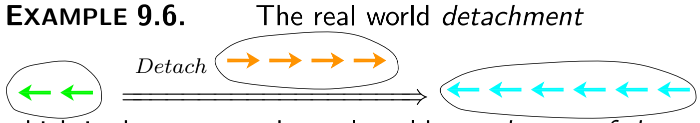
which is the same as the real world *attachment of the opposite*



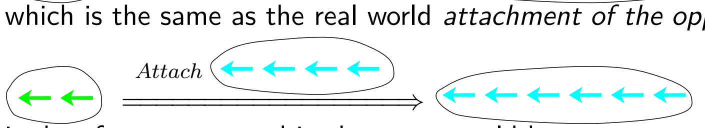
is therefore represented in the paper world by

$$\begin{aligned} +2\,\text{Arrows} \xrightarrow{\ominus +5\,\text{Arrows}}\; & +2\,\text{Arrows} \ominus +5\,\text{Arrows} \\ &= +2\,\text{Arrows} \oplus -5\,\text{Arrows} \\ &= -(5-2)\,\text{Arrows} \\ &= -3\,\text{Arrows} \end{aligned}$$

Example 9.6. The real world *detachment*



which is the same as the real world *attachment of the opposite*



is therefore represented in the paper world by

$$\begin{aligned} -2\,\text{Arrows} \xrightarrow{\ominus +4\,\text{Arrows}}\; & -2\,\text{Arrows} \ominus +4\,\text{Arrows} \\ &= -2\,\text{Arrows} \oplus -4\,\text{Arrows} \\ &= -(2+4)\,\text{Arrows} \\ &= -6\,\text{Arrows} \end{aligned}$$

Here are a few examples that focus on the numerators:

state
balance
in the black
in the red
action
initial state
final state
deposit
withdrawal
change

Example 9.9.1. In order to get $(+3) \ominus (+5)$,

i. We compute $(+3) \oplus$ Opposite $(+5)$ that is we compute $(+3) \oplus (-5)$
ii. Which, using the Chapter on signed addition, gives -2 .

Example 9.9.2. In order to get $(+3) \ominus (-5)$,

i. We compute $(+3) \oplus$ Opposite (-5) that is we compute $(+3) \oplus (+5)$
ii. Which, using the Chapter on signed addition, gives $+8$.

Example 9.9.3. In order to get $(-3) \ominus (+5)$,

i. We compute $(-3) \oplus$ Opposite $(+5)$ that is we compute $(-3) \oplus (-5)$
ii. Which, using the Chapter on signed addition, gives -8.

Example 9.9.4. In order to get $(-3) \ominus (-5)$,

i. We compute $(-3) \oplus$ Opposite (-5) that is we compute $(-3) \oplus (+5)$
ii. Which, using the Chapter on signed addition, gives $+2$.

9.3 Bank Accounts

A very important application of signed numerators is that they allow us to monitor real world bank accounts.

1. The standard terminology for monitoring bank accounts is as follows:
a. The real world **state** of a bank account at any time is represented on paper by a *signed* numerator called the **balance** and:

- If the balance is *positive*, we say that the real world bank account is **in the black** (Ahead of the game.).
- If the numerator which represents the balance is *negative*, we say that the real world bank account is **in the red** (In the hole).

b. A real world **action** on a bank account changes the bank account from an **initial state** to a **final state**.

$$\text{Initial State} \xRightarrow{\textit{Action}} \text{Final State}$$

An action on a bank account can be either a **deposit** represented on paper by a positive numerator or a **withdrawal** represented on paper by a negative numerator.
c. The **change** from an initial state to a final state is:
i. the final state,
from which we detach
ii. the initial state.
The reason is that each state is the result of *all prior* actions from the very beginning. So, by subtracting the *initial* state from the *final state*, we eliminate the effect of all the actions that resulted in the *initial* state leaving only the last one, namely the effect of the *last action*.

So, the change between an initial state and a final state is represented on paper by the *signed difference* gain loss

$$\text{final balance} \ominus \text{initial balance}$$

2. The *change* from an initial state to a final state can be:

- *up* in which case the change is called a **gain** and, on paper, the *difference* which represents a gain is a *positive* numerator.

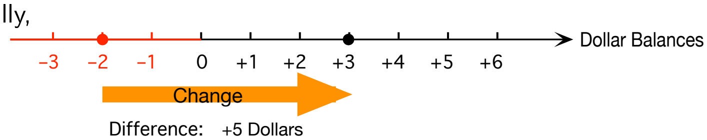
EXAMPLE 9.9.5.

– On Monday, Jill's balance was TWO ***dollars*** in the *red*
– On Wednesday, Jill's balance was THREE ***dollars*** in the *black*

So, from Monday to Wednesday, Jill's balance went *up* by FIVE ***dollars*** and, correspondingly, on paper the *difference* which represent the *change* is

$$\begin{aligned} \text{Difference} &= \textbf{Final Balance} \ominus \textbf{Initial Balance} \\ &= +3 \textbf{ Dollars} \ominus -2 \textbf{ Dollars} \\ &= +3 \textbf{ Dollars} \oplus +2 \textbf{ Dollars} \\ &= +5 \textbf{ Dollars} \end{aligned}$$

Graphically,



or can be

- *down* in which case the change is called a **loss** and, on paper, the *difference* which represents a loss is a *negative* numerator,

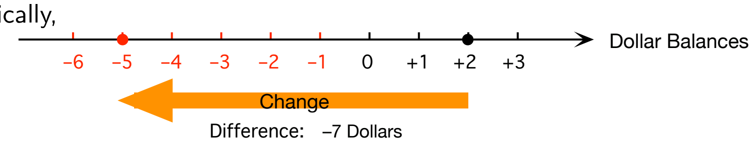
EXAMPLE 9.9.6.

– On Monday, Jack's balance was TWO ***dollars*** in the *black*
– On Wednesday, Jack's balance was FIVE ***dollars*** in the *red*

So, from Monday to Wednesday, Jack's balance went *down* by SEVEN ***dollars*** and, correspondingly, on paper the *difference* which represent the *change* is

$$\begin{aligned} \text{Difference} &= \textbf{Final Balance} \ominus \textbf{Initial Balance} \\ &= -5 \textbf{ Dollars} \ominus +2 \textbf{ Dollars} \\ &= -5 \textbf{ Dollars} \oplus -2 \textbf{ Dollars} \\ &= -7 \textbf{ Dollars} \end{aligned}$$

Graphically,



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