Chapter 2

Introduction in Dimension Two

As we go from Dimension One to Dimension Two, most everything will remain the same but we will encounter one major difference. In fact, the two-dimensional case is a good prototype for any higher dimensions.

Lists and Spaces

Because we will be dealing here with more than one number phrase at a time, we will have to use lists of number phrases which we will enclose between square brackets and which we will call \textit{baskets}.

\textbf{Example 1.} The list $[3 \text{Apples}, 17 \text{Bananas}]$ is a \textit{basket}.

Just like number phrases often come associated with a co-number phrase \textit{baskets} often come associated with lists of co-number phrases which we will call \textit{price-lists} and which we will write vertically.

\textbf{Example 2.} A \textit{basket} such as $[3 \text{Apples, 5 Bananas}]$ could come associated with a \textit{price-list} such as $\left[ \begin{array}{c} 7 \text{Cents} \\ \text{Apple} \\ 4 \text{Cents} \\ \text{Banana} \end{array} \right]$

Then, we will say that

\begin{itemize}
  \item All possible \textit{baskets} make up a \textit{space of baskets}
  \item All possible \textit{price-lists} make up a \textit{space of price-lists}
\end{itemize}

Finally, in order to picture spaces, we will use \textit{polygons}.
Value and Co-multiplication

The cost of a given number of items at a given unit-price is the number phrase whose numerator is the number of items multiplied by the unit-price and whose denominator is the “currency” in the unit-price.

**Example 3.** The value of 3 Apples at

The value of a given basket at a given price-list is then given by an operation, called co-multiplication, in which we multiply the numerator of the number phrases in the basket by the numerators of the co-number phrases in the price-list and add-up. The denominator of the result is the common “currency” in the co-number phrases in the price-list. We will use the symbol \( \odot \) for co-multiplication.

**Example 4.**

\[
\text{VALUE of } [3 \text{ Apples, 5 Bananas}] \text{ at } \begin{bmatrix} 7 \text{ Cents Apple}\n4 \text{ Cents Banana} \end{bmatrix} = [3 \text{ Apples, 5 Bananas}] \odot \begin{bmatrix} 7 \text{ Cents Apple}\n4 \text{ Cents Banana} \end{bmatrix}
\]

\[
= 3 \text{ Apples} \times \frac{7 \text{ Cents}}{\text{Apple}} + 5 \text{ Bananas} \times \frac{4 \text{ Cents}}{\text{Banana}}
\]

\[
= 3 \times 7 \text{ Cents} + 5 \times 4 \text{ Cents}
\]

\[
= 21 \text{ Cents} + 20 \text{ Cents}
\]

\[
= 41 \text{ Cents}
\]

Finally, we will call the set of all possible values the field of values and we have the following picture:

Valuation Function

An evaluation function is a function which, given a basket, outputs the value of that basket for a given price-list:

\[
\text{BASKET} \xrightarrow{\text{EVALUATION}|_{\text{PRICE-LIST}}} \text{EVALUATION}|_{\text{PRICE-LIST}}(\text{BASKET}) = \text{VALUE}
\]

**Example 5.**
The reverse problem for the function VALUATION is, given a PRICE-LIST, and given a VALUE, to determine what BASKETS, if any, we can get at that VALUE.

**Example 6.**

We have:

\[
\text{VALUATION} \left[ \begin{array}{c} 7 \\ 4 \end{array} \right] (x \text{ Apples}, y \text{ Bananas}) = 41 \text{ Cents}
\]

So the BASKETS that we can get with 41 Cents are given by the solutions of the equation

\[
7x + 4y = 41
\]